# Powers of Binomials

## Explore Expanding Binomials

🔄 **Online Activity** Use interactive tool to complete the Explore.

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❓ **INQUIRY** How can you use Pascal's triangle to write expansions of binomials?

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## Learn Powers of Binomials

You can expand binomials by following a set of rules and using patterns.

### Key Concept • Binomial Expansion

In the binomial expansion of $(a + b)^n$,

- there are $n + 1$ terms.
- $n$ is the exponent of $a$ in the first term and $b$ in the last term.
- in successive terms, the exponent of $a$ decreases by 1, and the exponent of $b$ increases by 1.
- the sum of the exponents in each term is $n$.
- the coefficients are symmetric.

**Pascal's triangle** is a triangle of numbers in which a row represents the coefficients of an expanded binomial $(a + b)^n$. Each row begins and ends with 1. Each coefficient can be found by adding the two coefficients above it in the previous row.

Instead of writing out the rows of Pascal's triangle, you can use the Binomial Theorem to expand a binomial. The Binomial Theorem uses combinations to calculate the coefficients of the binomial expansion.

### Key Concept • Binomial Theorem

If $n$ is a natural number, then $(a + b)^n =$

$$_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + {}_nC_3a^{n-3}b^3 + \ldots + {}_nC_na^0b^n$$

or

$$1a^nb^0 + \frac{n!}{1!(n-1)!}a^{n-1}b^1 + \frac{n!}{2!(n-2)!}a^{n-2}b^2 + \frac{n!}{3!(n-3)!}a^{n-3}b^3 + \ldots + 1a^0b^n$$

## Example 1 Use Pascal's Triangle

**Use Pascal's triangle to expand $(x + y)^7$.**

```
                    1                        (x + y)⁰
                1       1                    (x + y)¹
            1       2       1                (x + y)²
        1       3       3       1            (x + y)³
     1      4       6       4       1        (x + y)⁴
  1      5      10      10       5      1     (x + y)⁵
  1      6      15      20      15      6      1
1     7      21      35      35      21      7      1
```

$$(x + y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

### Today's Goal
- Expand powers of binomials by using Pascal's Triangle and the Binomial Theorem.

### Today's Vocabulary
Pascal's triangle

### 💭 Think About It!
Both $_nC_0$ and $_nC_n$ equal 1. What does this mean for the terms of a binomial expansion? How does this relate to Pascal's triangle?

Sample answer: The first and last terms will have a coefficient of 1. Similarly, each row in Pascal's triangle begins and ends with a 1, which represents the coefficients of the first and last terms of a binomial expansion.

### Study Tip

**Combinations** Recall that $_nC_r$ refers to the number of ways to choose $r$ objects from $n$ distinct objects. In the Binomial Theorem, $n$ is the exponent of $(a + b)^n$, and $r$ is the exponent of $b$ in each term. To calculate the coefficients, remember that $n!$ represents $n$ factorial. This is the product of all counting numbers beginning with $n$ and counting backward to 1. For example, $3! = 3 \cdot 2 \cdot 1$.

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## Check

Write the expansion of $(c + d)^4$.  $c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4$

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## 🌐 Example 2  Use the Binomial Theorem

**BASEBALL  In 2016, the Chicago Cubs won the World Series for the first time in 108 years. During the regular season, the Cubs played the Atlanta Braves 6 times, winning 3 games and losing 3 games. If the Cubs were as likely to win as to lose, find the probability of this outcome by expanding $(w + \ell)^6$.**

$(w + \ell)^6$

$= {}_6C_0 w^6 + {}_6C_1 w^5\ell + {}_6C_2 w^4\ell^2 + {}_6C_3 w^3\ell^3 + {}_6C_4 w^2\ell^4 + {}_6C_5 w\ell^5 + {}_6C_6 \ell^6$

$= w^6 + \dfrac{6!}{5!}w^5\ell + \dfrac{6!}{2!4!}w^4\ell^2 + \dfrac{6!}{3!3!}w^3\ell^3 + \dfrac{6!}{4!2!}w^2\ell^4 + \dfrac{6!}{5!}w\ell^5 + \ell^6$

$= w^6 + 6w^5\ell + 15w^4\ell^2 + 20w^3\ell^3 + 15w^2\ell^4 + 6w\ell^5 + \ell^6$

By adding the coefficients, you can determine that there were 64 combinations of wins and losses that could have occurred.

$20w^3\ell^3$ represents the number of combinations of 3 wins and

3 losses. Therefore, there was a $\dfrac{20}{64}$ or about a ___31___% chance of the

Cubs winning 3 games and losing 3 games against the Braves.

## Check

**GAME SHOW  A group of 8 contestants are selected from the audience of a television game show. If there are an equal number of men and women in the audience, find the probability of the contestants being 5 women and 3 men by expanding $(w + m)^8$. Round to the nearest percent if necessary.**  ___22___%

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## Example 3  Coefficients Other Than 1

**Expand $(2c - 6d)^4$.**

$(2c - 6d)^4$

$= {}_4C_0(2c)^4 + {}_4C_1(2c)^3(-6d) + {}_4C_2(2c)^2(-6d)^2 + {}_4C_3(2c)(-6d)^3 + {}_4C_4(-6d)^4$

$= 16c^4 + \dfrac{4!}{3!}(8c^3)(-6d) + \dfrac{4!}{2!2!}(4c^2)(36d^2) + \dfrac{4!}{3!}(2c)(-216d^3) + 1296d^4$

$= 16c^4 - 192c^3d + 864c^2d^2 - 1728cd^3 + 1296d^4$

🌐 **Go Online**  You can complete an Extra Example online.

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### 💬 Talk About It!

Describe a shortcut you could use to write out rows of Pascal's triangle instead of adding to find every number in a row. Explain your reasoning.

Sample answer: Since the coefficients in an expanded binomial are symmetric, you can just find the first or last half of the numbers in a row. Then, complete the row by using the same numbers.

### Study Tip

**Assumptions**  To use the Binomial Theorem, we assumed that the teams had an equal chance of winning and losing. Although teams are not always evenly matched and may benefit from a home field advantage, assuming there is an equal chance of either event occurring allows us to reasonably estimate the probability of an outcome.

### Study Tip

**Coefficients**  When the binomial to be expanded has coefficients other than 1, the coefficients will no longer be symmetric. In these cases, it may be easier to use the Binomial Theorem.

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