Dividing Polynomials

Explore Using Algebra Tiles to Divide Polynomials

Online Activity Use algebra tiles to complete the Explore.

INQUIRY How can you use a model to divide polynomials?

Learn Dividing Polynomials by Using Long Division

To divide a polynomial by a monomial, find the quotient of each term of the polynomial and the monomial.

$$\frac{6x^2 - 15x}{3x} = \frac{6x^2}{3x} - \frac{15x}{3x} = 2x - 5$$

You can divide a polynomial by a polynomial with more than one term by using a process similar to long division of real numbers. This process is known as the Division Algorithm. The resulting quotient may be a polynomial or a polynomial with a remainder.

Key Concept • Division Algorithm

Words: If $f(x)$ and $g(x)$ are two polynomials in which $g(x) \neq 0$ and the degree of $g(x)$ is less than the degree of $f(x)$, then there exists a unique quotient $q(x)$ and a unique remainder $r(x)$ such that $f(x) = q(x)g(x) + r(x)$, where the remainder is zero or the degree of $r(x)$ is less than the degree of $g(x)$.

Symbols: $\frac{f(x)}{g(x)} = q(x) \rightarrow f(x) = q(x)g(x)$

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \rightarrow f(x) = q(x)g(x) + r(x)$$

Example: $\frac{2x^2 - 5x - 3}{x - 3} = 2x + 1$

$$2x^2 - 5x - 3 = (x - 3)(2x + 1)$$
$$= 2x^2 - 5x - 3$$

$$\frac{x^2 - 4x - 1}{x + 1} = x - 5 + \frac{4}{x + 1}$$

$$x^2 - 4x - 1 = (x + 1)(x - 5) + 4$$
$$= x^2 - 4x - 5 + 4$$
$$= x^2 - 4x - 1$$

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Today's Goals
- Divide polynomials by using long division.
- Divide polynomials by using synthetic division.

Today's Vocabulary
synthetic division

Talk About It!
Is the following statement *always*, *sometimes*, or *never* true? Justify your argument.

If a quadratic polynomial is divided by a binomial with a remainder of 0, the binomial is a factor of the polynomial.

Always; sample answer: If the binomial divides the polynomial with a remainder of zero, the quotient is another binomial. The product of the two binomials is the original polynomial.

Study Tip
In algebra, *unique* means *only one*. So, there is only one quotient and remainder that will satisfy the Division Algorithm for each polynomial.

Your Notes

Think About It!

How can you check the solution?

Sample answer: Multiply $x + 4$ and $x - 9$ and check that the product is $x^2 - 5x - 36$.

Watch Out!

Signs Remember that you are subtracting throughout the process of long division. Carefully label the signs of the coefficients to avoid a sign error.

Go Online

You can learn how to divide polynomials by using long division by watching the video online.

Example 1 Divide a Polynomial by a Monomial

Find $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$.

$(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$

$= \dfrac{24a^4b^3 + 18a^2b^2 - 30ab^3}{6ab}$ — Write a fraction.

$= \dfrac{24a^4b^3}{6ab} + \dfrac{18a^2b^2}{6ab} - \dfrac{30ab^3}{6ab}$ — Sum of quotients

$= \dfrac{24}{6}a^{4-1}b^{3-1} + \dfrac{18}{6}a^{2-1}b^{2-1} - \dfrac{30}{6}a^{1-1}b^{3-1}$ — Divide.

$= \underline{4a^3b^2} + 3ab - \underline{5b^2}$ — Simplify.

Check

Find $(9x^9y^5 + 21x^4y^4 - 12x^3y^2) \div (3x^2y^2)$. $\underline{3x^7y^3 + 7x^2y^2 - 4x}$

Example 2 Divide a Polynomial by a Binomial

Find $(x^2 - 5x - 36) \div (x + 4)$.

$$
\begin{array}{r}
x - 9 \\
x + 4 \overline{)\, x^2 - 5x - 36} \\
(-)\, x^2 + 4x \phantom{{}-36} \\
\hline
-9x - 36 \\
(-)\, -9x - 36 \\
\hline
0
\end{array}
$$

The quotient is $\underline{x - 9}$ and the remainder is $\underline{0}$.

Check

Find $\dfrac{x^2 + 6x - 112}{x - 8}$. $\underline{x + 14}$

Example 3 Find a Quotient with a Remainder

Find $\dfrac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.

$$
\begin{array}{r}
3z^2 + z - 2 \\
z - 5 \overline{)\, 3z^3 - 14z^2 - 7z + 3} \\
(-)\, 3z^3 - 15z^2 \phantom{{}-7z+3} \\
\hline
z^2 - 7z \phantom{{}+3} \\
(-)\, z^2 - 5z \phantom{{}+3} \\
\hline
-2z + \underline{3} \\
(-)\, -2z + 10 \\
\hline
-7
\end{array}
$$

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The quotient is $3z^2 + z - 2$ and the remainder is -7.

Therefore, $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5} = 3z^2 + z - 2 - \frac{7}{z - 5}$.

Check

Find the quotient of $(-4x^3 + 5x^2 - 2x - 9)(x - 2)^{-1}$.

$-4x^2 + 3x + 4 - \frac{1}{x-2}$

Learn Dividing Polynomials by Using Synthetic Division

Synthetic division is an alternate method used to divide a polynomial by a binomial of degree 1. You may find this to be a quicker, simpler method.

Key Concept • Synthetic Division

Step 1 After writing a polynomial in standard form, write the coefficients of the dividend. If the dividend is missing a term, use 0 as a placeholder. Write the constant a of the divisor $x - a$ in the box. Bring the first coefficient down.

Step 2 Multiply the number just written in the bottom row by a, and write the product under the next coefficient.

Step 3 Add the product and the coefficient above it.

Step 4 Repeat **Steps 2** and **3** until you reach a sum in the last column.

Step 5 Write the quotient. The numbers along the bottom row are the coefficients of the quotient. The power of the first term is one less than the degree of the dividend. The final number is the remainder.

Example 4 Use Synthetic Division

Find $(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.

Step 1 Write the coefficients of the dividend and write the constant a in the box. Because $x + 3 = x - (-3)$, $a = -3$. Then bring the first coefficient down.

$$\begin{array}{r|rrr|r} \underline{-3} & 3 & -2 & -53 & -60 \\ & \downarrow & & & \\ \hline & 3 & & & \end{array}$$

Step 2 Multiply by a and write the product. The product of the coefficient and a is $3(-3) = -9$.

$$\begin{array}{r|rrr|r} \underline{-3} & 3 & -2 & -53 & -60 \\ & & -9 & & \\ \hline & 3 & & & \end{array}$$

Step 3 Add the product and the coefficient.

$$\begin{array}{r|rrr|r} \underline{-3} & 3 & -2 & -53 & -60 \\ & & -9 & & \\ \hline & 3 & -11 & & \end{array}$$

Step 4 Repeat **Steps 2** and **3** until you reach a sum in the last column.

$$\begin{array}{r|rrr|r} \underline{-3} & 3 & -2 & -53 & -60 \\ & & -9 & 33 & 60 \\ \hline & 3 & -11 & -20 & 0 \end{array}$$

(continued on the next page)

Think About It!

Describe a method you could use to check your answer.

Sample answer: I can use long division to check that the quotient is the same.

Watch Out!

Missing terms Add placeholders for terms that are missing from the polynomial. In this case, there are 0 x^3-terms.

Step 5 Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2. The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is $3x^2 - 11x - 20$.

Example 5 Divisor with Coefficient Other Than 1

Find $\dfrac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$.

To use synthetic division, the lead coefficient of the divisor must be 1.

$$\frac{(4x^4 - 37x^2 + 4x + 9) \div 2}{(2x - 1) \div 2}$$ Divide the numerator and denominator by 2.

$$= \frac{2x^4 - \frac{37}{2}x^2 + 2x + \frac{9}{2}}{x - \frac{1}{2}}$$ Simplify the numerator and denominator.

$x - a = x - \frac{1}{2}$, so $a = \frac{1}{2}$.

Complete the synthetic division.

$\frac{1}{2}$	2	0	$-\frac{37}{2}$	2	$\frac{9}{2}$
		1	$\frac{1}{2}$	-9	$-\frac{7}{2}$
	2	1	-18	-7	1

The resulting expression is $2x^3 + x^2 - 18x - 7 + \frac{1}{x - \frac{1}{2}}$. Now simplify the fraction.

$$\frac{1}{x - \frac{1}{2}} = \frac{(1)2}{\left(x - \frac{1}{2}\right) \cdot 2}$$ Multiply the numerator and denominator by 2.

$$= \frac{2}{2x - 1}$$ Simplify.

The solution is $2x^3 + x^2 - 18x - 7 + \frac{2}{2x - 1}$.

You can check your answer by using long division.

Check

Find $(4x^4 + 3x^3 - 12x^2 - x + 6)(4x + 3)^{-1}$.

$x^3 - 3x + 2$

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