Operations with Polynomials

Learn Adding and Subtracting Polynomials

A polynomial is a monomial or the sum of two or more monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. The degree of a polynomial is the greatest degree of any term in the polynomial.

Polynomials can be added or subtracted by performing the operations indicated and combining like terms. You can subtract a polynomial by adding its additive inverse.

The sum or difference of polynomials will have the same variables and exponents as the original polynomials, but possibly different coefficients. Thus, the sum or difference of two polynomials is also a polynomial.

A set is **closed** if and only if an operation on any two elements of the set produces another element of the same set. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is closed under the operations of addition and subtraction.

Example 1 Identify Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $x^6 + \sqrt[3]{x} - 4$

This expression is not a polynomial because $\sqrt[3]{x}$ is not a monomial.

b. $5a^4b + 3a^2b^7 - 9$

This expression is a polynomial because each term is a monomial. The degree of the first term is $4 + 1$ or 5, the degree of the second term is $2 + 7$ or 9, and the degree of the third term is 0. So, the degree of the polynomial is 9.

c. $\frac{2}{3}x^{-5} - 6x^{-3} - x$

The expression is not a polynomial because x^{-5} and x^{-3} are not monomials.

Check

State the degree of each polynomial.

A. $x^7 + 6x^5 - \frac{1}{3}$ 7

B. $3c^7d^2 + 5cd - 9$ 9

C. p^{10} 10

D. 25 0

Go Online You can complete an Extra Example online.

Today's Goals

- Add and subtract polynomials.
- Multiply polynomials.

Today's Vocabulary

binomial

trinomial

closed

FOIL method

Think About It!

Identify the like terms in $8x^5 + 3x^2 - 10$ and $4x^3 - 7x^2 + 5$.

$3x^2$ and $-7x^2$; -10 and 5

Study Tip

Degree 0 and 1
Remember that constant terms have a degree of 0 and variable terms with no exponent indicated have a degree of 1.

Go Online

You can learn how to add and subtract polynomials by watching the video online.

Your Notes

Talk About It!

Notice that the terms in the sum have the same variables and exponents as the terms of the original expressions: x^3, x^2, x, and a constant. Will this always be the case? Explain.

Sample answer: If the sum of any of the like terms is 0, then that degree term will not appear in the resulting polynomial.

Example 2 Add Polynomials

Find $(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$.

Method 1 Add horizontally.

Group and combine like terms.

$(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$

$= (6x^3 + x^3) + (7x^2 - 4x^2) + (-2x - 8x) + (5 + 1)$ Group like terms.

$= \underline{7}x^3 + \underline{3}x^2 - \underline{10}x + \underline{6}$ Combine like terms.

Method 2 Add vertically.

Align like terms vertically and add.

$$\begin{array}{r} 6x^3 + 7x^2 - 2x + 5 \\ (+)\ x^3 - 4x^2 - 8x + 1 \\ \hline 7x^3 + 3x^2 - 10x + 6 \end{array}$$

Align like terms.

Combine like terms.

Check

Find $(2x^3 + 9x^2 + 6x - 3) + (4x^3 - 7x^2 + 5x)$. $6x^3 + 2x^2 + 11x - 3$

Example 3 Subtract Polynomials

Find $(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$.

Method 1 Subtract horizontally.

Group and combine like terms.

$(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$ Original equation

$= 2x^5 + 11x^4 + 7x - 8 - 5x^4 - 9x^3 + 3x - 4$ Distribute -1.

$= 2x^5 + (11x^4 - 5x^4) + (-9x^3) + (7x + 3x) + (-8 - 4)$ Group like terms.

$= 2x^5 + 6x^4 - 9x^3 + 10x - 12$ Combine like terms.

Method 2 Subtract vertically.

Align like terms vertically and subtract.

$$\begin{array}{r} 2x^5 + 11x^4 + 0x^3 + 7x - 8 \\ (-)\ 0x^5 + 5x^4 + 9x^3 - 3x + 4 \\ \hline 2x^5 + 6x^4 - 9x^3 + 10x - 12 \end{array}$$

Check

Find $(8x^2 - 3x + 1) - (5x^3 + 2x^2 - 6x - 9)$. $-5x^3 + 6x^2 + 3x + 10$

Think About It!

Why is it helpful to insert placeholders for the $0x^3$ and $0x^5$ terms? Why were placeholders not included for the x^2-term in either polynomial?

Sample answer: Since neither polynomial has an x^2-term, the like terms can be aligned without adding a placeholder to each expression.

Study Tip

Additive Inverse

Distributing -1 to each term in the polynomial being subtracted is the same as finding and adding its additive inverse.

Explore Multiplying Polynomials

Online Activity Use a table to complete the Explore.

INQUIRY How is using a table to multiply polynomials related to the Distributive Property?

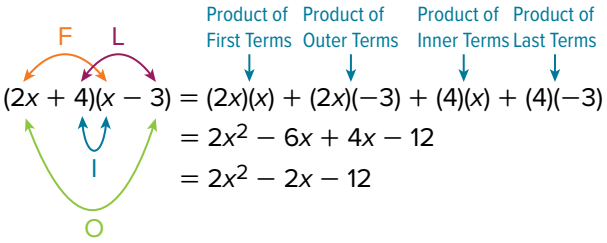
Learn Multiplying Polynomials

Polynomials can be multiplied by using the Distributive Property to multiply each term in one polynomial by each term in the other. When polynomials are multiplied, the product is also a polynomial. Therefore, the set of polynomials is closed under the multiplication. This is similar to the system of integers, which is also closed under multiplication. To multiply two binomials, you can use a shortcut called the **FOIL method**.

Key Concept • FOIL Method

Words: Find the sum of the products of ***F*** the *First* terms, ***O*** the *Outer* terms, ***I*** the *Inner* terms, and ***L*** the *Last* terms.

Symbols:



Example 4 Simplify by Using the Distributive Property

Find $2x(4x^3 + 5x^2 - x - 7)$.

$2x(4x^3 + 5x^2 - x - 7) = 2x(4x^3) + 2x(\underline{5x^2}) + 2x(-x) + 2x(\underline{-7})$

$= \underline{8}\,x^4 + 10x^3 - \underline{2}\,x^2 - 14x$

Example 5 Multiply Binomials

Find $(3a + 5)(a - 7)(4a + 1)$.

Step 1 Multiply any two binomials.

$(3a + 5)(a - 7) = 3a(\underline{a}) + 3a(\underline{-7}) + 5(a) + 5(-7)$ FOIL Method

$= 3a^2 - 21a + \underline{5}\,a - \underline{35}$ Multiply.

$= 3a^2 - \underline{16}\,a - 35$ Combine like terms.

Step 2 Multiply the result by the remaining binomial.

$(3a^2 - 16a - 35)(4a + 1)$

$= 3a^2(4a + 1) + (\underline{-16a})(4a + 1) + (\underline{-35})(4a + 1)$

$= \underline{12a^3} + 3a^2 - 64a^2 - 16a - 140a - 35$

$= 12a^3 - \underline{61a^2} - 156a \underline{-35}$

Check

Find $(-2r - 3)(5r - 1)(r + 4)$. $-10r^3 - 53r^2 - 49r + 12$

Go Online You can complete an Extra Example online.

Think About It!

Why are the exponents added when you multiply the monomials?

Sample answer: Because the monomials have the same base *x*, their exponents are added by using the Product of Powers Property.

Go Online

for an example of how to multiply two trinomials.

Think About It!

What does x represent in the polynomial expression for the volume of the cake?

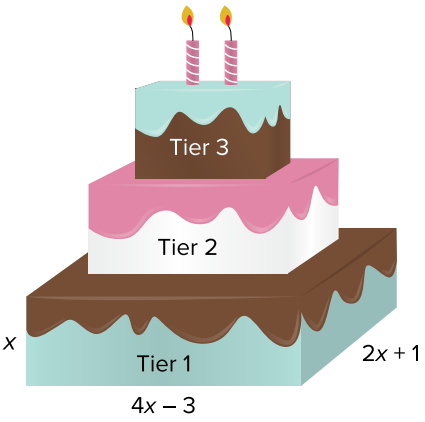
the height of tier 1

Problem-Solving Tip

Solve a Simpler Problem Some complicated problems can be more easily solved by breaking them into several simpler problems. In this case, finding the volume of each tier individually simplifies the situation and makes finding the total volume easier.

Apply Example 6 Write and Simplify a Polynomial Expression

BAKING **Byron is baking a three-tier cake for a birthday party. Each tier will have $\frac{1}{2}$ the volume of the previous tier. The dimensions of the first tier are shown. Find the total volume of the cake.**



1 What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to find the total volume of the cake, which is the sum of all 3 tiers. How can I represent the volume of each tier as a polynomial? Which properties will I need to know? I can find the answers to my questions by referencing other examples in the lesson.

2 How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will find and simplify the volume of each tier and then add them together. I will use the Distributive Property and FOIL method to complete the task.

3 What is your solution?

Use your strategy to solve the problem.

What is the volume of each tier?

Tier 1: $8x^3 - 2x^2 - 3x$, Tier 2: $4x^3 - x^2 - 1.5x$,
Tier 3: $2x^3 - 0.5x^2 - 0.75x$

What is the total volume of the cake?

$14x^3 - 3.5x^2 - 5.25x$

4 How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: Because all the expressions are based on the expression for the volume of Tier 1, I can check that the expression for Tier 1 is correct. I can factor the expression for volume of Tier 1 to ensure that the factors are the same as the given dimensions.

$8x^3 - 2x^2 - 3x$	Expression for Tier 1
$= (8x^2 - 2x - 3)x$	Factor x from each term.
$= (4x - 3)(2x + 1)x$ ✓	Factor $8x^2 - 2x - 3$.