Polynomial Functions

Explore Power Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY How do the coefficient and degree of a function of the form $f(x) = ax^n$ affect its end behavior?

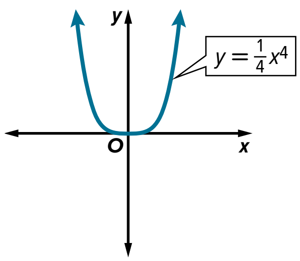
Learn Graphing Power Functions

A **power function** is any function of the form $f(x) = ax^n$ where a and n are nonzero real numbers. For a power function, a is the **leading coefficient** and n is the **degree**, which is the value of the exponent. A power function with positive integer n is called a **monomial function**.

Key Concept • End Behavior of a Monomial Function

Degree: even
Leading Coefficient: positive
End Behavior:

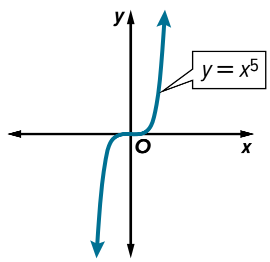
As $x \to -\infty$, $f(x) \to \infty$.
As $x \to \infty$, $f(x) \to \infty$.



Domain: all real numbers
Range: all real numbers ≥ 0

Degree: odd
Leading Coefficient: positive
End Behavior:

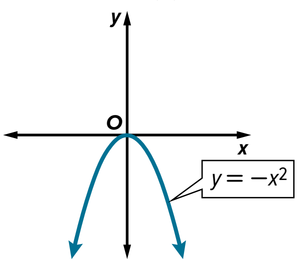
As $x \to -\infty$, $f(x) \to -\infty$.
As $x \to \infty$, $f(x) \to \infty$.



Domain: all real numbers
Range: all real numbers

Degree: even
Leading Coefficient: negative
End Behavior:

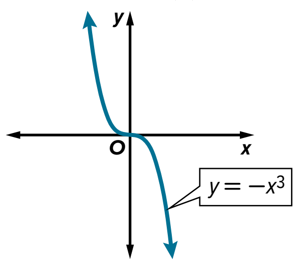
As $x \to -\infty$, $f(x) \to -\infty$.
As $x \to \infty$, $f(x) \to -\infty$.



Domain: all real numbers
Range: all real numbers ≤ 0

Degree: odd
Leading Coefficient: negative
End Behavior:

As $x \to -\infty$, $f(x) \to \infty$.
As $x \to \infty$, $f(x) \to -\infty$.



Domain: all real numbers
Range: all real numbers

Today's Goals

- Graph and analyze power functions.
- Graph and analyze polynomial functions.

Today's Vocabulary

power function
leading coefficient
degree
monomial function
polynomial in one variable
standard form of a polynomial
degree of a polynomial
polynomial function
quartic function
quintic function

Talk About It!

Is $f(x) = \sqrt{x}$ a power function? a monomial function? Explain your reasoning.

Sample answer: $f(x) = \sqrt{x}$ is a power function because it is equivalent to $f(x) = x^{\frac{1}{2}}$, and $\frac{1}{2}$ is a nonzero constant real number. It is not a monomial function because $\frac{1}{2}$ is not a positive integer.

Your Notes

Key Concept • Zeros of Even and Odd Degree Functions

Odd-degree functions will always have at least one real zero. Even-degree functions may have any number of real zeros or no real zeros at all.

Example 1 End Behavior and Degree of a Monomial Function

Describe the end behavior of $f(x) = -2x^3$ using the leading coefficient and degree, and state the domain and range.

The leading coefficient of $f(x)$ is $\underline{-2}$, which is negative.

The degree is $\underline{3}$, which is odd.

Because the leading coefficient is negative and the degree is odd, the end behavior is that as $x \to -\infty$, $f(x) \to \underline{\infty}$ and as $x \to \infty$, $f(x) \to \underline{-\infty}$.

Because the leading coefficient is negative and the degree is odd, the domain and range are <u>all real numbers</u>.

Check

Describe the end behavior, domain, and range of $f(x) = -10x^6$.

end behavior: As $x \to -\infty$, $f(x) \to \underline{-\infty}$ and as $x \to \infty$, $f(x) \to \underline{-\infty}$.

domain: <u>all real numbers</u> range: <u>all real numbers ≤ 0</u>

Go Online You can watch a video to see how to graph power functions on a TI-84.

Example 2 Graph a Power Function by Using a Table

PRESSURE **For water to flow through a garden hose at a certain rate in gallons per minute (gpm), it needs to have a specific pressure in pounds per square inch (psi). Through testing and measurement, a company that produces garden hoses determines that the pressure P given the flow rate F is defined by $P(F) = \frac{3}{2}F^2$. Graph the function $P(F)$, and state the domain and range.**

Think About It! Interpret the domain and range given the context of the situation.

Sample answer: Because it is unreasonable to have negative flow rate or negative pressure from a garden hose, both the domain and range are constricted to values greater than or equal to zero.

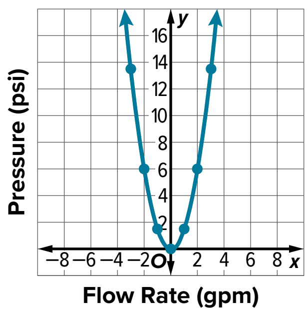
Steps 1 and 2 Find a and n. Then state the domain and range.

For $P(F) = \frac{3}{2}F^2$, $a = \underline{\frac{3}{2}}$, and $n = \underline{2}$.

Because a is positive and n is even, the domain is <u>all real numbers</u> and the range is all real numbers $\underline{\geq 0}$.

Steps 3–5 Create a table of values and graph the ordered pairs.

F	$\frac{3}{2}F^2$	$P(F)$
-2	$\frac{3}{2}(-2)^2$	6
-1	$\frac{3}{2}(-1)^2$	1.5
0	$\frac{3}{2}(0)^2$	0
1	$\frac{3}{2}(1)^2$	1.5
2	$\frac{3}{2}(2)^2$	6



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Explore Polynomial Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY How is the degree of a function related to the number of times its graph intersects the x-axis?

Learn Graphing Polynomial Functions

A **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1}, a_1, and a_0 are real numbers, and n is a nonnegative integer. Because the terms are written in order from greatest to least degree, this polynomial is written in **standard form**. The **degree of a polynomial** is n and the leading coefficient is a_n.

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable. You have learned about constant, linear, quadratic, and cubic functions. A **quartic function** is a fourth-degree function. A **quintic function** is a fifth-degree function. The degree tells you the maximum number of times that the graph of a polynomial function intersects the x-axis.

Example 3 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $2x^4 - 3x^3 - 4x^2 - 5x + 6a$ degree: 4 leading coefficient: 2

b. $7x^3 - 2$ degree: 3 leading coefficient: 7

c. $4x^2 - 2xy + 8y^2$ This is not a polynomial in one variable. There are two variables, x and y.

d. $x^5 + 12x^4 - 3x^3 + 2x^2 + 8x + 4$ degree: 5 leading coefficient: 1

Check

Select the degree and leading coefficient of $11x^3 + 5x^2 - 7x - \frac{6}{x}$. D

A. degree: 3, leading coefficient: 11

B. degree: 11, leading coefficient: 3

C. This is not a polynomial in one variable. There are two variables, x and y.

D. This is not a polynomial in one variable. The term $\frac{6}{x}$ has the variable with an exponent less than 0.

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Go Online
You can learn how to graph a polynomial function by watching the video online.

Think About It!
If a polynomial function has a leading coefficient of 4, can you determine its end behavior? Explain your reasoning.

No; sample answer: I would need to know the degree of the function to determine its end behavior.

Think About It!
Jamison says the leading coefficient of $4x^2 - 3 + 2x^3 - x$ is 4. Do you agree or disagree? Justify your reasoning.

Disagree; sample answer: The term with the greatest degree is $2x^3$, and the coefficient of that term is 2.

Watch Out!
Leading Coefficients
If the term with the greatest degree has no coefficient shown, as in part **d**, the leading coefficient is 1.

Think About It!

What values of x make sense in the context of the situation? Justify your reasoning.

Sample answer: Because x is a percent, only values between 0 and 1, inclusively, make sense in the context of the situation.

Study Tip

Axes Labels Notice that the x-axis is measuring the percent of the radius, not the actual length of the radius.

Example 4 Evaluate and Graph a Polynomial Function

SUN The density of the Sun, in grams per centimeter cubed, expressed as a percent of the distance from the core of the Sun to its surface can be modeled by the function $f(x) = 519x^4 - 1630x^3 + 1844x^2 + 155$, where x represents the percent as a decimal. At the core $x = 0$, and at the surface $x = 1$.

Part A Evaluate the function.

Find the core density of the Sun at a radius 60% of the way to the surface.

Because we need to find the core density at a radius 60% of the way to the surface, $x = 0.6$. So, replace x with 0.6 and simplify.

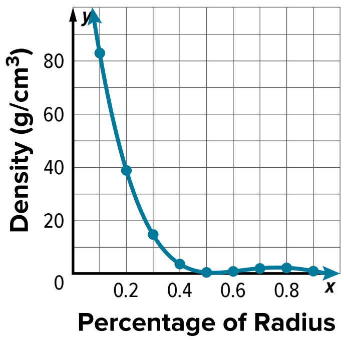
$$f(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$
$$= 519(0.6)^4 - 1630(0.6)^3 + 1844(0.6)^2 - 889(0.6) + 155$$
$$= 67.2624 - 352.08 + 663.84 - 533.4 + 155$$
$$= 0.6224 \ \frac{\text{g}}{\text{cm}^3}$$

Part B Graph the function.

Sketch a graph of the function.

Substitute values of x to create a table of values. Then plot the points, and connect them with a smooth curve.

x	$f(x)$
0.1	82.9619
0.2	38.7504
0.3	14.4539
0.4	3.4064
0.5	0.1875
0.7	1.7819
0.8	1.9824
0.9	0.7859



Check

CARDIOLOGY To help predict heart attacks, doctors can inject a concentration of dye in a vein near the heart to measure the cardiac output in patients. In a normal heart, the change in the concentration of dye can be modeled by $f(x) = -0.006x^4 + 0.140x^3 - 0.053x^2 + 1.79x$, where x is the time in seconds.

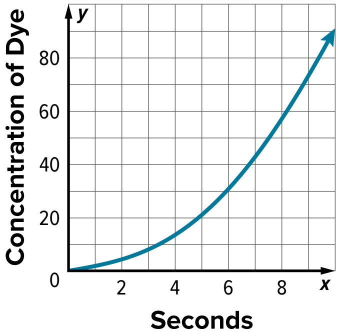
Part A Find the concentration of dye after 5 seconds.

$f(5) =$ 21.375

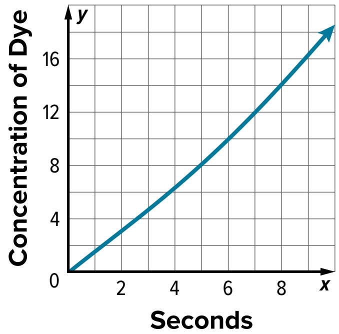
Go Online You can complete an Extra Example online.

Part B Select the graph of the concentration of dye over 10 seconds. A

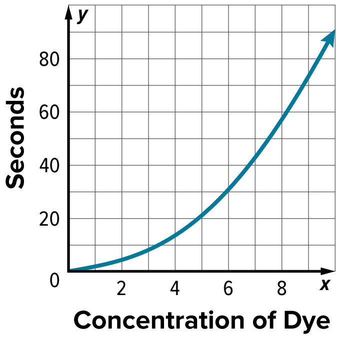
A.



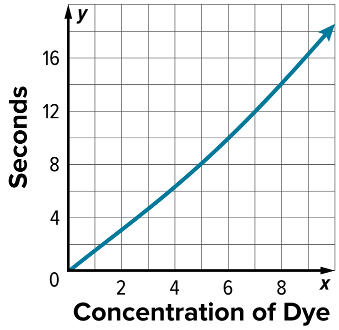
B.



C.

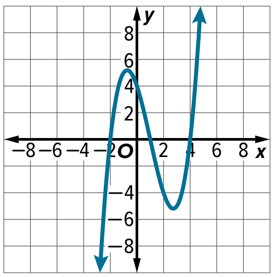


D.



Example 5 Zeros of a Polynomial Function

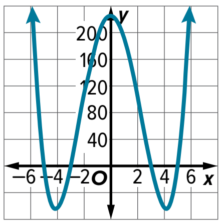
Use the graph to state the number of real zeros of the function.



The real zeros occur at $x = -2$, 1, and 4, so there are three real zeros.

Check

Use the graph to state the number of real zeros of the function.



The function has 4 real zero(s).

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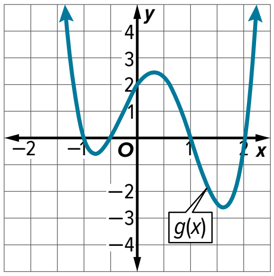
Study Tip

Zeros The real zeros occur at values of x where $f(x) = 0$, or where the polynomial intersects the x-axis. Recall that odd-degree polynomial functions have at least one real zero and even-degree polynomial functions have any number of real zeros. So, the minimum number of times that an odd-degree polynomial intersects the x-axis is 1, and the minimum number of times that an even-degree polynomial intersects the x-axis is 0.

Example 6 Compare Polynomial Functions

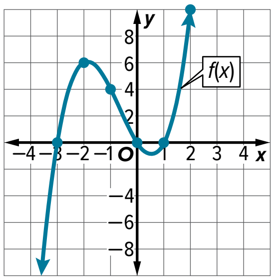
Examine $f(x) = x^3 + 2x^2 - 3x$ and $g(x)$ shown in the graph.



Part A Graph $f(x)$.

Substitute values for x to create a table of values. Then plot the points, and connect them with a smooth curve.

x	$f(x)$
−3	0
−2	6
−1	4
0	0
1	0
2	10
3	36



Part B Analyze the extrema.

Which function has the greater relative maximum?

$f(x)$ has a relative maximum at approximately $y = 6$, and $g(x)$ has a relative maximum between $y = 2$ and $y = 3$. So, $f(x)$ has the greater relative maximum.

Part C Analyze the key features.

Compare the zeros, x- and y-intercepts, and end behavior of $f(x)$ and $g(x)$.

zeros:

$f(x)$: −3, 0, 1

$g(x)$: The graph appears to intersect the x-axis at −1, −0.5, 1, 2

intercepts:

$f(x)$: x-intercepts: −3, 0, 1; y-intercept: 0

$g(x)$: x-intercepts: −1, −0.5, 1, 2; y-intercept: 2

end behavior:

$f(x)$: As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$.

$g(x)$: As $x \to -\infty$, $g(x) \to \infty$, and as $x \to \infty$, $g(x) \to \infty$.

Think About It!

Find the domain and range of $f(x)$. Does $g(x)$ have the same domain and range? Explain.

The domain and range of $f(x)$ are both all real numbers. No; sample answer: The domain of $g(x)$ is also all real numbers, but the range is all real numbers greater than or equal to the minimum, which is between $y = -2$ and $y = -3$.

Study Tip

Zeros The zeros of a polynomial function are the x-coordinates of the points at which the graph intersects the x-axis.

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

Record your observations here.

See students' observations.

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