Quadratic Inequalities

Explore Graphing Quadratic Inequalities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you represent a quadratic inequality graphically?

Learn Graphing Quadratic Inequalities

You can graph quadratic inequalities in two variables by using the same techniques used to graph linear inequalities in two variables. A **quadratic inequality** is an inequality of the form $y > ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y < ax^2 + bx + c$, or $y \leq ax^2 + bx + c$.

Key Concept • Graphing Quadratic Inequalities

Step 1 Graph the related function.
Step 2 Test a point not on the parabola.
Step 3 Shade accordingly.

Example 1 Graph a Quadratic Inequality ($<$ or $\leq$)

Graph $y \leq x^2 - 2x + 8$.

Step 1 Graph the related function.

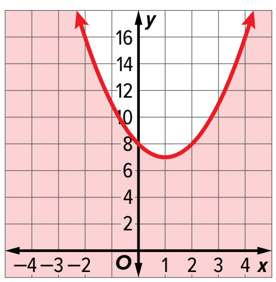
Because the inequality is less than or equal to, the parabola should be solid.

Step 2 Test a point not on the parabola.

$y \leq x^2 - 2x + 8$	Original inequality
$0 \overset{?}{\leq} (0)^2 - 2(0) + 8$	$(x, y) = (0, 0)$
$\underline{0} \leq \underline{8}$	True

Shade the region that contains the point.

Step 3 Shade accordingly.



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Today's Goals
- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in two variables by graphing.

Today's Vocabulary
quadratic inequality

Think About It!
How do you know whether to make the parabola solid or dashed?

Sample answer: If the inequality contains $<$ or $>$, then it is dashed. If it contains $\leq$ or $\geq$, then it is solid.

Study Tip
(0, 0) If (0, 0) is not a point on the parabola, then it is often the easiest point to test when determining which part of the graph to shade.

Your Notes

Example 2 Graph a Quadratic Inequality (> or ≥)

Graph $y > -5x^2 + 10x$.

Step 1 Graph the related function.

Because the inequality is greater than, the parabola should be dashed.

Step 2 Test a point not on the parabola.

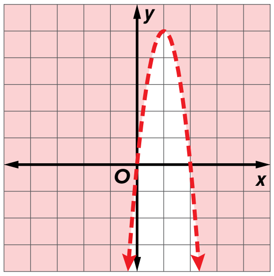
Because (0, 0) is on the parabola, use (1, 0) as a test point.

$y > -5x^2 + 10x$	Original inequality
$0 \overset{?}{>} -5(1)^2 + 10(1)$	$(x, y) = (1, 0)$
$\underline{0} > \underline{5}$	False

So, (1, 0) is not a solution of the inequality.

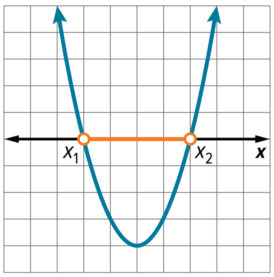
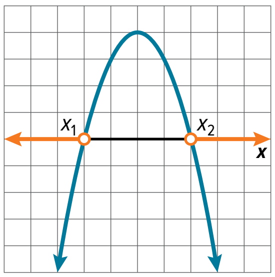
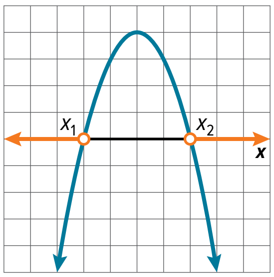
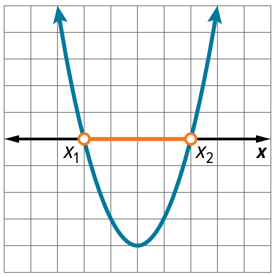
Step 3 Shade accordingly.

Because (1, 0) is not a solution of the inequality, shade the region that does not contain the point.



Learn Solving Quadratic Inequalities

Key Concept • Solving Quadratic Inequalities

$ax^2 + bx + c < 0$ Graph $y = ax^2 + bx + c$ and identify the x-values for which the graph lies *below* the x-axis. For ≤, include the x-intercepts in the solution.	$a > 0$  $\{x \mid x_1 < x < x_2\}$	$a < 0$  $\{x \mid x < x_1 \text{ or } x > x_2\}$
$ax^2 + bx + c > 0$ Graph $y = ax^2 + bx + c$ and identify the x-values for which the graph lies *above* the x-axis. For ≥, include the x-intercepts in the solution.	$a > 0$  $\{x \mid x < x_1 \text{ or } x > x_2\}$	$a < 0$  $\{x \mid x_1 < x < x_2\}$

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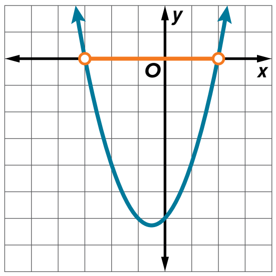
Example 3 Solve a Quadratic Inequality ($<$ or $\leq$) by Graphing

Solve $x^2 + x - 6 < 0$ by graphing.

Because the quadratic expression is less than 0, the solution consists of x-values for which the graph of the related function lies *below* the x-axis. Begin by finding the zeros of the related function.

$x^2 + x - 6 = 0$	Related equation
$(x - \underline{2})(x + \underline{3}) = 0$	Factor.
$x = \underline{2}$ or $x = \underline{-3}$	Zero Product Property

Sketch the graph of a parabola that has x-intercepts at 2 and -3. The graph should open up because $a > 0$.



The graph lies below the x-axis between $\underline{-3}$ and $\underline{2}$. Thus, the solution set is $\{x \mid \underline{-3} < x < \underline{2}\}$ or in interval notation $(\underline{-3,\ 2})$.

Think About It!

How could you check your solution?

Sample answer: I could test a point on the segment, as well as one point to the left and one point to the right of the segment.

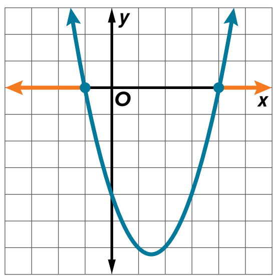
Example 4 Solve a Quadratic Inequality ($>$ or $\geq$) by Graphing

Solve $x^2 - 3x - 4 \geq 0$ by graphing.

Because the quadratic expression is greater than or equal to 0, the solution consists of x-values for which the graph of the related function lies *on* and *above* the x-axis. Begin by finding the zeros of the related function.

$x^2 - 3x - 4 = 0$	Related equation
$(x - \underline{4})(x + \underline{1}) = 0$	Factor.
$x = \underline{4}$ or $x = \underline{-1}$	Zero Product Property

Sketch the graph of a parabola that has x-intercepts at –1 and 4. The graph should open up because $a > 0$.



The graph lies above and on the x-axis when $x \leq \underline{-1}$ or $x \geq \underline{4}$. Thus, the solution set is $\{x \mid x \leq \underline{-1}$ or $x \geq \underline{4}\}$ or $(\underline{-\infty, -1}] \cup [\underline{4}, \underline{\infty})$.

Talk About It

For a quadratic inequality of the form $ax^2 + bx + c > 0$ where $a < 0$, if the related equation has no real roots, what is the solution set? Explain your reasoning.

Sample answer: The solution set is the empty set, because there are no values of x for which $ax^2 + bx + c > 0$.

Check

Solve $-\frac{1}{4}x^2 + x + 1 > 0$ by graphing and write the solution set.

$\{x \mid \underline{-0.83} < x < \underline{4.83}\}$

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Example 5 Solve a Quadratic Inequality Algebraically

GARDENING **Marcus is planning a garden. He has enough soil to cover 104 square feet, and wants the dimensions of the garden to be at least 5 feet by 10 feet. If he wants to increase the length and width by the same number of feet, by what value can he increase the dimensions of the garden without needing to buy more soil? Create a quadratic inequality and solve it algebraically.**

Step 1 Determine the quadratic inequality.

$A = \ell w$	Area formula
$= (x + 10)(x + 5)$	$\ell = x + 10$; $w + 5$
$= x^2 + \underline{15x} + 50$	FOIL and simplify.

The area must be less than or equal to 104 square feet,

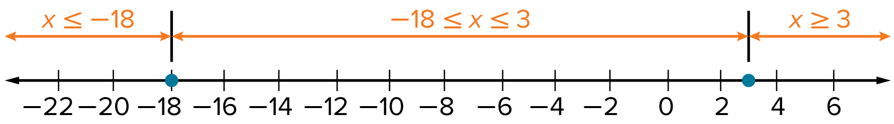
so $x^2 + 15x + 50 \leq \underline{104}$.

Step 2 Solve the related equation.

$x^2 + 15x + 50 = 104$	Related equation
$x^2 + 15x \underline{- 54} = 0$	Subtract 104 from each side.
$(x \underline{+ 18})(x \underline{- 3}) = 0$	Factor.
$x = -18$ or $x = 3$	Zero Product Property

Steps 3 and 4 Plot the solutions on a number line and test a value from each interval.

Use dots because −18 and 3 are solutions of the original inequality.



Test a value from each interval to see if it satisfies the original inequality.

Test $x = 20$, $x = 0$, and $x = 5$. The only value that satisfies the original inequality is $x = \underline{0}$, so the solution set is $[-18, \underline{3}]$. So, Marcus can increase the length and width up to $\underline{3}$ feet without needing the buy more soil. The interval $-18 \leq x \leq 0$ is not relevant because Marcus does not want to decrease the length and width or leave it as is.

Check

MANUFACTURING An electronics manufacturer can model their profits in dollars P when they sell x video players by using the function $P(x) = -0.1x^2 + 75x - 1000$. How many video players can they sell so they make \$7500 or less?

The company will make \$7500 or less if they make $\underline{139}$ video players or fewer and/or $\underline{611}$ video players or more.

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