Using the Quadratic Formula and the Discriminant

Today's Goals
- Solve quadratic equations by using the Quadratic Formula.
- Determine the number and type of roots of a quadratic equation.

Today's Vocabulary
discriminant

Learn Using the Quadratic Formula

To solve any quadratic equation, you can use the Quadratic Formula.

Key Concept • Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Go Online You can see how the Quadratic Formula is derived.

Example 1 Real Roots, c is Positive

CONTEST **At the World Championship Punkin Chunkin contest in Bridgeville, Delaware, pumpkins are launched hundreds of yards. The path of a pumpkin can be modeled by $h = -4.9t^2 + 11.7t + 42$, where h is the height and t is the number of seconds after launch.**

Part A Use the Quadratic Formula to solve $0 = -4.9t^2 + 11.7t + 42$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-11.7 \pm \sqrt{(11.7)^2 - 4(-4.9)(42)}}{2(-4.9)} \qquad a = -4.9,\ b = 11.7,\ c = 42$$

$$= \frac{-11.7 \pm \sqrt{136.89 + 823.2}}{-9.8} \qquad \text{Square and multiply.}$$

$$= \frac{-11.7 \pm \sqrt{960.09}}{-9.8} \qquad \text{Add.}$$

$$= \frac{11.7 + \sqrt{960.09}}{9.8} \text{ or } \frac{11.7 - \sqrt{960.09}}{9.8} \qquad \text{Multiply by } \frac{-1}{-1}.$$

The approximate solutions are 4.4 seconds and −2.0 seconds.

Part B Interpret the roots.

The negative root does not make sense in this context because the pumpkin launches at 0 seconds. The pumpkin lands after 4.4 seconds.

Think About It!

Why are the roots not at 0 and 4.4 seconds, when the pumpkin is launched and when it lands?

Sample answer: The pumpkin was launched from a starting height of 42 feet, so it was not at 0 feet when $t = 0$.

Go Online You can complete an Extra Example online.

Check

DIVING A diver jumps from a diving board that is 10 feet high, and she wants to figure out how far from the board she is before she enters the water. Her arc can be modeled by $y = -4.9x^2 + 2.5x + 10$, where y is her height in meters and x is time in seconds.

Part A Solve $0 = -4.9x^2 + 2.5x + 10$. $\frac{2.5 + \sqrt{202.25}}{9.8}$, $\frac{2.5 - \sqrt{202.25}}{9.8}$

Part B Interpret the roots.

The diver enters the water after approximately 1.7 seconds.

Your Notes

Example 2 Real Roots, c is Negative

Solve $x^2 + 4x - 17 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-17)}}{2(1)}$$ $a = 1, b = 4, c = -17$

$$= \frac{-4 \pm \sqrt{84}}{2}$$ Simplify.

$$= \frac{-4 \pm \sqrt{4} \cdot \sqrt{21}}{2}$$ Product Property of Square Roots

$$= \frac{-4 \pm 2\sqrt{21}}{2}$$ $\sqrt{4} = 2$

$$= -2 \pm \sqrt{21}$$ Divide the numerator and denominator by 2.

Check

Solve $3x^2 - 5x - 1 = 0$ by using the Quadratic Formula.

$\frac{5 \pm \sqrt{37}}{6}$

Example 3 Complex Roots

Solve $5x^2 + 8x + 11 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-8 \pm \sqrt{8^2 - 4(5)(11)}}{2(5)}$$ $a = 5, b = 8, c = 11$

$$= \frac{-8 \pm \sqrt{-156}}{10}$$ Simplify.

$$= \frac{-8 \pm \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{39}}{10}$$ Product Property of Square Roots

$$= \frac{-8 \pm 2i\sqrt{39}}{10}$$ Write as a complex number.

$$= \frac{-4 \pm i\sqrt{39}}{5}$$ Divide the numerator and denominator by 2.

Check

Solve $9x^2 - 3x + 18 = 0$ by using the Quadratic Formula.

$\frac{1 \pm i\sqrt{71}}{6}$

Go Online You can complete an Extra Example online.

Explore The Discriminant

Online Activity Use graphing technology to complete the Explore.

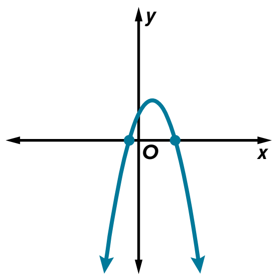
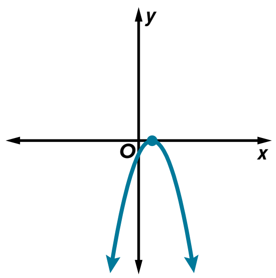
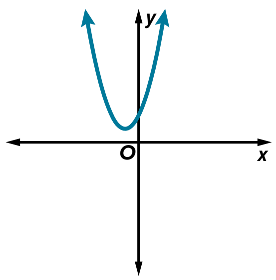
INQUIRY How does the discriminant of a quadratic equation relate to its roots?

Learn Using the Discriminant

In the Quadratic Formula, the **discriminant** is the expression under the radical sign, $b^2 - 4ac$. The value of the discriminant can be used to determine the number and type of roots of a quadratic equation.

Key Concept • Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	**Type and Number of Roots**	**Example of Graph of Related Function**
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Talk About It

Why are the roots of a quadratic equation complex if the discriminant is negative?

Sample answer: If the discriminant is negative, then the radical expression in the Quadratic Formula will include the imaginary unit $i = \sqrt{-1}$. Thus, the roots will be complex numbers because they contain i.

Go Online You can complete an Extra Example online.

Think About It!

Is it possible for a quadratic equation to have zero real or complex roots?

No; sample answer: if the discriminant is 0, then there is exactly one solution. If it is not zero, there are two solutions.

Go Online to practice what you've learned in Lessons 3-2 and 3-4 through 3-6.

Example 4 The Discriminant, Real Roots

Examine $2x^2 - 10x + 7 = 0$.

Part A Find the value of the discriminant for $2x^2 - 10x + 7 = 0$.

$a = $ 2 $\qquad b = $ −10 $\qquad c = $ 7

$b^2 - 4ac = (-10)^2 - 4(2)(7)$

$= 100 - 56$

$= 44$

Part B Describe the number and type of roots for the equation.

The discriminant is nonzero, so there are two roots. The discriminant is positive and not a perfect square, so the roots are irrational.

Check

Examine $2x^2 + 8x + 8 = 0$.

Part A Find the value of the discriminant for $2x^2 + 8x + 8 = 0$.

$b^2 - 4ac = $ 0

Part B Describe the number and type of roots for the equation.

There is/are 1 rational root(s).

Example 5 The Discriminant, Complex Roots

Examine $-5x^2 + 10x - 15 = 0$.

Part A Find the value of the discriminant for $-5x^2 + 10x - 15 = 0$.

$a = $ −5 $\qquad b = $ 10 $\qquad c = $ −15

$b^2 - 4ac = (10)^2 - 4(-5)(-15)$

$= 100 - 300$

$= -200$

Part B Describe the number and type of roots for the equation.

The discriminant is nonzero, so there are two roots. The discriminant is negative, so the roots are complex.

Check

Examine $10x^2 - 4x + 7 = 0$.

Part A Find the value of the discriminant for $10x^2 - 4x + 7 = 0$.

$b^2 - 4ac = $ −264

Part B Describe the number and type of roots for the equation.

There is/are 2 complex root(s).

Go Online You can complete an Extra Example online.