Solve Quadratic Equations by Factoring

Explore Finding the Solutions of Quadratic Equations by Factoring

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use factoring to solve a quadratic equation?

Learn Solving Quadratic Equations by Factoring

The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$, where $a \neq 0$. In this equation, p and q represent the x-intercepts of the graph of the related function. For example, $0 = x^2 - 2x - 3$ can be written in the factored form $0 = (x - 3)(x + 1)$ and its related graph has x-intercepts of -1 and 3.

Key Concepts • Factoring	
Using the Distributive Property	$ax + bx = x(a + b)$
Factoring Trinomials	$x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$

Key Concept • Zero Product Property

Words: For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and $b = 0$.

Example: If $(x - 2)(x + 4) = 0$, then $x - 2 = 0$, $x + 4 = 0$, or both $x - 2 = 0$ and $x + 4 = 0$.

To solve a quadratic equation by factoring, first make sure that one side of the equation is 0, and factor the trinomial. Use the Zero Product Property to write separate equations. Then use the properties of equality to isolate the variable.

Example 1 Factor by Using the Distributive Property

Solve $12x^2 - 2x = x$ by factoring. Check your solution.

$12x^2 - 2x = x$	Original equation
$12x^2 - \underline{3}x = 0$	Subtract x from each side.
$3x(\underline{4x}) - 3x(\underline{1}) = 0$	Factor the GCF.
$\underline{3x}(4x - 1) = 0$	Distributive Property
$3x = 0$ or $4x - 1 = 0$	Zero Product Property
$x = \underline{0}$ $\quad x = \underline{\frac{1}{4}}$	Solve.

Go Online You can complete an Extra Example online.

Today's Goals
- Solve quadratic equations by factoring.
- Solve quadratic equations by factoring special products.

Today's Vocabulary
- factored form
- difference of squares
- perfect square trinomials

Think About It!

The equation $x^2 - 2x - 3 = 0$ could be solved by factoring, where $x^2 - 2x - 3 = (x - 3)(x + 1)$. How are the factors of the equation related to the roots, or zeros, of the related function $f(x) = x^2 - 2x - 3$?

Setting each factor equal to 0 and solving the resulting equations results in the zeros of the related function. So, $f(3) = 0$ and $f(-1) = 0$.

Go Online You can watch a video to see how to use algebra tiles to factor a polynomial using the Distributive Property.

Your Notes

Think About It!
Choose two integers and write an equation in standard form with these roots. How would the equation change if the signs of the two roots were switched?

Sample answer: 2 and 5; $x^2 - 7x + 10 = 0$; -2 and -5; $x^2 + 7x + 10 = 0$; the linear term is the only term that changes signs.

Example 2 Factor a Trinomial

Solve $x^2 - 6x - 9 = 18$ by factoring. Check your solution.

$x^2 - 6x - 9 = 18$	Original equation
$x^2 - 6x - \underline{27} = 0$	Subtract 18 from each side.
$(x + \underline{3})(x - \underline{9}) = 0$	Factor the trinomial.
$x + 3 = 0$ or $x - 9 = 0$	Zero Product Property
$x = \underline{-3}$ $\quad x = \underline{9}$	Solve.

Example 3 Solve an Equation by Factoring

ACCELERATION **The equation $d = vt + \frac{1}{2}at^2$ represents the displacement d of a car traveling at an initial velocity v where the acceleration a is constant over a given time t. Find how long it takes a car to accelerate from 30 mph to 45 mph if the car moved 605 feet and accelerated slowly at a rate of 2 feet per second squared.**

1 What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: Solve the equation to find the time for the car to accelerate. The acceleration is given in feet per second squared and the velocity is given in miles per hour. How do I address the difference in units?

2 How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: Convert the velocity to feet per second. Then substitute the distance, velocity, and acceleration into the formula and solve for time.

3 What is your solution?

Use your strategy to solve the problem.

What is the velocity in feet per second? 44 fps

How long it takes the car to accelerate from 30 mph to 45 mph? 11 s

4 How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Sample answer: The solutions of the equation are -55 and 11. Because time cannot be negative, $t = 11$ is the only viable solution in the context of the situation.

Go Online You can complete an Extra Example online.

Check

SALES A clothing store is analyzing their market to determine the profitability of their new dress design. If $P(x) = -16x^2 + 1712x - 44{,}640$ represents the store's profit when x is the price of each dress, find the prices at which the store makes no profit on the design. B

A. $11.25 and $15.50

B. $45 and $62

C. $50 and $54

D. $180 and $248

Example 4 Factor a Trinomial Where a is Not 1

Solve $3x^2 + 5x + 15 = 17$ by factoring. Check your solution.

$3x^2 + 5x + 15 = 17$	Original equation
$3x^2 + 5x - 2 = 0$	Subtract 17 from each side.
$(3x - 1)(x + 2) = 0$	Factor the trinomial.
$3x - 1 = 0$ or $x + 2 = 0$	Zero Product Property
$x = \frac{1}{3}$ $\quad x = -2$	Solve.

Check

Solve $4x^2 + 12x - 27 = 13$ by factoring. Check your solution.

$x =$ −5, 2

Learn Solving Quadratic Equations by Factoring Special Products

Key Concept • Factoring Differences of Squares

Words: To factor $a^2 - b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Symbols: $a^2 - b^2 = (a + b)(a - b)$

Key Concept • Factoring Perfect Square Trinomials

Words: To factor $a^2 + 2ab + b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Symbols: $a^2 + 2ab + b^2 = (a + b)^2$

If a is positive and b is negative, then $a^2 - 2ab + b^2 = (a - b)^2$.

Not all quadratic equations have solutions that are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$. For example, you know that the solution of $x^2 = -4$ must be complex because there is no real number for which its square is -4. If you take the square root of each side, $x = 2i$ or $-2i$.

Think About It!

Explain how to determine which values should be chosen for m and p when factoring a polynomial of the form $ax^2 + bx + c$.

Sample answer: Find two numbers, m and p, with a product of ac and a sum of b.

Math History Minute

English mathematician and astronomer **Thomas Harriot (1560–1621)** was one of the first, if not the first, to consider the imaginary roots of equations. Harriot advanced the notation system for algebra and studied negative and imaginary numbers.

Example 5 Factor a Difference of Squares

Solve $81 = x^2$ by factoring. Check your solution.

$81 = x^2$	Original equation
$81 - x^2 = 0$	Subtract x^2 from each side.
$9^2 - x^2 = 0$	Write in the form $a^2 - b^2$.
$(\underline{9} + \underline{x})(\underline{9} - \underline{x}) = 0$	Factor the difference of squares.
$9 + x = 0$ or $9 - x = 0$	Zero Product Property
$x = \underline{-9}$ $\quad x = \underline{9}$	Solve.

Go Online You can watch a video to see how to use algebra tiles to factor a difference of squares.

Check

Solve $x^2 = 529$ by factoring. Check your solution.

$x = \underline{-23}, \underline{23}$

Example 6 Factor a Perfect Square Trinomial

Solve $16y^2 - 22y + 23 = 26y - 13$ by factoring. Check your solution.

$16y^2 - 22y + 23 = 26y - 13$	Original equation
$16y^2 - \underline{48}y + 23 = -13$	Subtract $26y$ from each side.
$16y^2 - 48y + \underline{36} = 0$	Add 13 to each side.
$(\underline{4y})^2 - 2(\underline{4y})(\underline{-6}) + (\underline{-6})^2 = 0$	Factor the perfect square trinomial.
$(\underline{4y} - \underline{6})^2 = 0$	Simplify.
$y = \underline{\frac{3}{2}}$	Take the square root of each side and solve.

Think About It!

Why does this equation have one solution instead of two?

Sample answer: The factor $4y - 6$ is repeated, so the equation only has one solution.

Check

Solve $16x^2 - 22x + 15 = 10x - 1$ by factoring. Check your solution.

$x = \underline{1}$

Example 7 Complex Solutions

Solve $x^2 = -144$ by factoring. Check your solution.

$x^2 = -144$	Original equation
$x^2 + 144 = 0$	Add 144 to each side.
$x^2 - (-144) = 0$	Write as a difference of squares.
$x^2 - (12i)^2 = 0$	$\sqrt{-144} = -12i$
$(x + 12i)(x - 12i) = 0$	Factor the difference of squares.
$x + 12i = 0$ or $x - 12i = 0$	Zero Product Property
$x = \underline{-12i}$ $\quad x = \underline{12i}$	Solve.

Go Online You can complete an Extra Example online.

Think About It!

Explain why both $(-12i)^2$ and $(12i)^2$ equal -144.

Sample answer: $(-12i)(-12i) = 144i^2$ or -144, and $(12i)(12i) = 144i^2$ or -144

Watch Out!

Complex Numbers Remember i^2 equals -1, not 1.