Solving Quadratic Equations by Graphing

Explore Roots of Quadratic Equations

🔗 **Online Activity** Use graphing technology to complete the Explore.

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> ⓠ **INQUIRY** How can you use the graph of a quadratic function to find the solutions of its related equation?

Learn Solving Quadratic Equations by Graphing

A **quadratic equation** is an equation that includes a quadratic expression.

Key Concept • Standard Form of a Quadratic Equation

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$ and a, b, and c are integers.

One method for finding the roots of a quadratic equation is to find the zeros of a related quadratic function. You can identify the solutions or roots of an equation by finding the x-intercepts of the graph of a related function. Often, exact roots cannot be found by graphing. You can estimate the solutions by finding the integers between which the zeros are located on the graph of the related function.

Example 1 One Real Solution

Solve $10 - x^2 = 4x + 14$ by graphing.

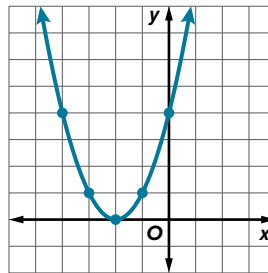Rearrange terms so that one side of the equation is 0.

$0 = x^2 + \underline{4} x + \underline{4}$

Find the axis of symmetry.

$x = -\frac{b}{2a} = -\frac{4}{2(1)}$ or $\underline{-2}$

Make a table of values, plot the points, and connect them with a curve.

x	y
−4	4
−3	1
−2	0
−1	1
0	4

The zero of the function is $\underline{-2}$. Therefore, the solution of the equation is $\underline{-2}$ or $\{x \mid x = \underline{-2}\}$.

Check

Solve $x^2 + 7x = 31x - 144$ by graphing. $x = \underline{12}$

Today's Goals
- Solve quadratic equations by graphing.

Today's Vocabulary
quadratic equation

standard form of a quadratic equation

💭 Think About It!

How can you determine the number of solutions of a quadratic equation?

Sample answer: Graph a related function and find the number of x-intercepts.

Study Tip

Solutions of Quadratic Equations A quadratic equation can have one real solution, two real solutions, or no real solutions.

💭 Think About It!

How can you find the solution of the equation from the table?

Sample answer: I can see from the table that −2 is the x-intercept of the related function, so −2 is a solution of the equation.

Example 2 Two Real Solutions

Use a quadratic equation to find two real numbers with a sum of 24 and a product of 143.

Let x represent one of the numbers. Then **24** $- x$ will represent the other number. So $x(24 - x) =$ **143**.

What do you need to find?
x and $24 - x$

Step 1 Solve the equation for 0.

$x(24 - x) = 143$ Original equation

$24x - x^2 = 143$ Distributive Property

$0 = x^2 - 24x + 143$ Subtract $24x - x^2$ from each side.

Step 2 Find the axis of symmetry.

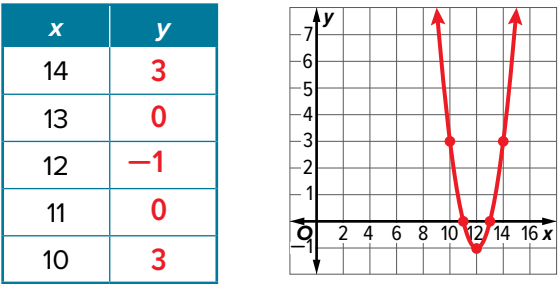$x = -\dfrac{b}{2a}$ Equation of the axis of symmetry

$x = -\dfrac{-24}{2(1)}$ $a = 1, b = -24$

$x =$ **12**   Simplify.

Step 3 Make a table of values and graph the function.

x	y
14	**3**
13	**0**
12	**−1**
11	**0**
10	**3**

Steps 4 and 5 Find the zero(s) and determine the solution.

The zeros of the function are **11** and **13**.

$x = 11$ or $x = 13$, so $24 - x = 13$ or $24 - x = 11$. Thus, the two numbers with a sum of 24 and a product of 143 are **11** and **13**.

Check

Use a quadratic equation to find two real numbers with a sum of −43 and a product of 306. **−9** and **−34**

▶ **Go Online** You can complete an Extra Example online.

Go Online You can watch a video to see how to solve quadratic equations by graphing on a graphing calculator.

💭 **Think About It!**

Explain why 9 and 15 cannot be solutions, even though their sum is 24.

Sample answer: The product of 9 and 15 is 135, not 143. Using them would result in a different equation with different solutions.

Example 3 Estimate Roots

Solve $-x^2 + 3x + 7 = 0$ by graphing. If the exact roots cannot be found, state the consecutive integers between which the roots are located.

Find the axis of symmetry. $x = -\dfrac{b}{2a} = -\dfrac{3}{2(-1)}$ or $= \dfrac{3}{2}$

Make a table of values, plot the points, and connect them with a curve.

The x-intercepts of the graph indicate that one solution is between __−2__ and __−1__, and the other solution is between __4__ and __5__.

x	y
−2	**−3**
−1	**3**
0	**7**
1	**9**
2	**9**
3	**7**
4	**3**
5	**−3**
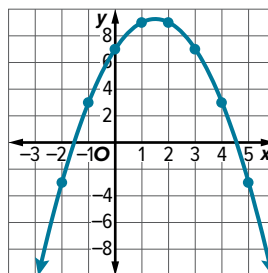
Check

Use a graph to find all of the solutions of $x^2 + 9x - 5 = 0$. Select all of the pairs of consecutive integers between which the roots are located.

between −10 and −9, between 0 and 1

Example 4 Solve by Using a Table

**Use a table to solve $-x^2 + 5x - 1 = 0$.**

Steps 1 and 2 Enter the function and view the table.

Enter $-x^2 + 5x - 1$ in the **Y=** list. Use the **TABLE** window to find where the sign of **Y1** changes. The sign changes between $x = 0$ and $x = $ __1__.

Steps 3 and 4 Edit the table settings and find a more accurate location.

Use **TBLSET** to change **ΔTbl** to 0.1 and look again for the sign change. Repeat this for 0.01 and 0.001 to get a more accurate location of one zero.

One zero is located at approximately $x = $ __0.209__
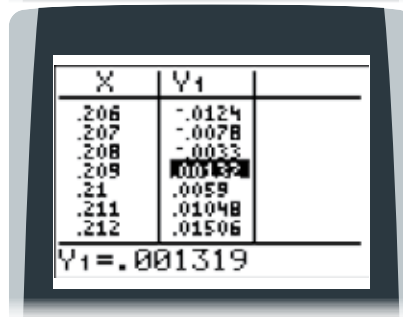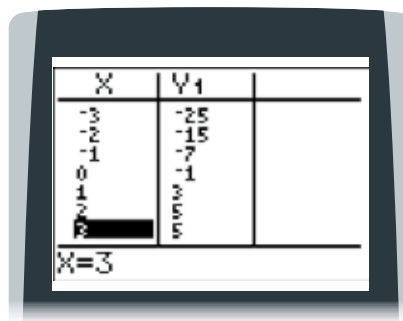
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Talk About It

How can you estimate the solutions of the equation from the table? Explain your reasoning.

Sample answer: I can find where the value of the function is positive for one value and negative for a second value. I know that there will be at least one zero between those two values.

Watch Out!

Graphing Calculator If you cannot see the graph of the function on your graphing calculator, you may need to adjust the viewing window. Having the proper viewing window will also make it easier to see the zeros.

Steps 5 and 6 Find the other zero determine the solutions of the equation.

Repeat the process to find the second zero of the function.

The zeros of the function are at approximately 0.209 and 4.791, so the solutions to the equation are approximately __0.209__ and __4.791__.

Check

Use a table to find all of the solutions of $-x^2 - 3x + 8 = 0$.

__−4.702__ and __1.702__

🌐 **Example 5** Solve by Using a Calculator

FOOTBALL **A kicker punts a football. The height of the ball after t seconds is given by $h(t) = -16t^2 + 50t + h_0$, where h_0 is the initial height. If the ball is 1.5 feet above the ground when the punter's foot meets the ball, how long will it take the ball to hit the ground?**

We know that h_0 is the initial height, so $h_0 = 1.5$. We need to find t when $h(t)$ is 0. Use a graphing calculator to graph the related function $h(t) = -16t^2 + 50t + \underline{1.5}$.

**Step 1** Enter the function in the **Y=** list, and press graph.

Step 2 Use the zero feature in the **CALC** menu to find the positive zero.

Step 3 Find the left bound by placing the cursor to the left of the intercept.

Step 4 Find the right bound.

Step 5 Find and interpret the solution.

The zero is approximately __3.15__. Thus, the ball hit the ground approximately __3.15__ seconds after it was punted.
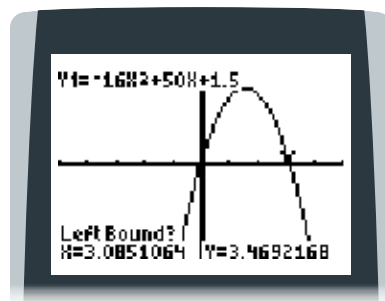
Check

SOCCER A goalie punts a soccer ball. If the ball is 1 foot above the ground when her foots meets the ball, find how long it will take, to the nearest hundredth of a second, for the ball to hit the ground. Use the formula $h(t) = -16t^2 + 35t + h_0$, where t is the time in seconds and h_0 is the initial height.

__2.22__ seconds

🔎 **Go Online** You can complete an Extra Example online.