

Optimization with Linear Programming

Learn Finding Maximum and Minimum Values

Linear programming is the process of finding the maximum or minimum values of a function for a region defined by a system of inequalities.

Key Concept • Linear Programming

- Step 1** Graph the inequalities.
- Step 2** Determine the coordinates of the vertices.
- Step 3** Evaluate the function at each vertex.
- Step 4** For a bounded region, determine the maximum and minimum. For an unbounded region, test other points within the feasible region to determine which vertex represents the maximum or minimum.

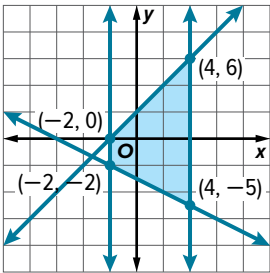
Example 1 Maximum and Minimum Values for a Bounded Region

Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$-2 \leq x \leq 4$
 $y \leq x + 2$
 $y \geq -0.5x - 3$
 $f(x, y) = -2x + 6y$

Steps 1 and 2 Graph the inequalities and determine the vertices.

The vertices of the feasible region are $(-2, -2)$, $(-2, 0)$, $(4, 6)$ and $(4, -5)$.



Step 3 Evaluate the function at each vertex.

(x, y)	$-2x + 6y$	$f(x, y)$
$(-2, -2)$	$-2(-2) + 6(-2)$	-8
$(-2, 0)$	$-2(-2) + 6(0)$	4
$(4, -5)$	$-2(4) + 6(-5)$	-38
$(4, 6)$	$-2(4) + 6(6)$	28

Step 4 Determine the maximum and minimum.

The maximum value is 28 at $(4, 6)$. The minimum value is -38 at $(4, -5)$.

Today's Goals

- Find maximum and minimum values of a function over a region.
- Solve real-world optimization problems by graphing systems of inequalities maximizing or minimizing constraints.

Today's Vocabulary

linear programming
optimization

Study Tip

Unbounded Regions

An unbounded feasible region does not necessarily contain a maximum or minimum. If it does, then it has either a maximum or a minimum, but not both.

Study Tip

Feasible Region To determine the feasible region, you can shade the solution set of each inequality individually, and then find where they all overlap. Shading each inequality using a different color or shading style can help you easily determine the feasible region.

Study Tip

Feasible Region To determine whether an unbounded region has a maximum or minimum for the function $f(x, y)$, you need to test several points in the feasible region to see if any values of $f(x, y)$ are greater than or less than the values of $f(x, y)$ for the vertices.

Think About It!

Why is it not possible for an unbounded feasible region to have both a maximum and a minimum?

Sample answer:
Because the feasible region is open, there are viable solutions that are either much greater or much less than the value of the vertices.

Example 2 Maximum and Minimum Values for an Unbounded Region

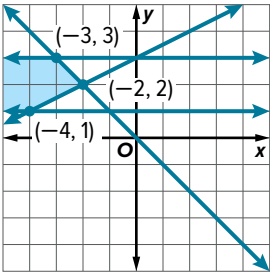
Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$1 \leq y \leq 3$
 $y \leq -x$
 $y \geq 0.5x + 3$
 $f(x, y) = -x + y$

Steps 1 and 2 Graph the inequalities and determine the vertices.

The vertices of the feasible region are $(-3, 3)$, $(-2, 2)$, $(-4, 1)$.

Notice that the region is **unbounded**. This may indicate that there is no minimum or maximum value.



Step 3 Evaluate the function.

Evaluate at each vertex and a point in the feasible region.

(x, y)	$-x + y$	$f(x, y)$
$(-3, 3)$	$-(-3) + 3$	6
$(-2, 2)$	$-(-2) + 2$	4
$(-4, 1)$	$-(-4) + 1$	5
$(-10, 2)$	$-(-10) + 2$	12

Step 4 The minimum value is **4** at $(-2, 2)$. There is **no** maximum value, because $(-10, 2)$ is a point in the feasible region and yields a greater $f(x, y)$ value than any of the vertices.

Explore Using Technology with Linear Programming

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use technology to find the maximum or minimum values of a function over a given region?

Go Online You can complete an Extra Example online.

Learn Linear Programming

Optimization is the process of seeking the optimal price or amount that is desired to minimize costs or maximize profits.

Key Concept • Optimization with Linear Programming

- Step 1** Define the variables.
- Step 2** Write a system of inequalities.
- Step 3** Graph the system of inequalities.
- Step 4** Find the coordinates of the vertices of the feasible region.
- Step 5** Write a linear function to be maximized or minimized.
- Step 6** Evaluate the function at each vertex by substituting the coordinates into the function.
- Step 7** Interpret the results.

Example 3 Optimizing with Linear Programming

GARDENING Avoree has a 30-square-foot plot in the school greenhouse and wants to plant lettuce and cucumbers while minimizing the amount of water she uses for them. Each cucumber requires 2.25 square feet of space and uses 25 gallons of water over the lifetime of the plant. Each lettuce plant requires 1.5 square feet of space and uses 17 gallons of water. She wants to grow at least 4 of each type of plant and at least 16 plants in total. Determine how many of each plant Avoree should plot in order to minimize her water usage.

Complete the table.

Vegetable	Minimum	Maximum	Water per Plant (gal)
cucumber	4	13	25
lettuce	4	20	17

Step 1 Define the variables.

Because the number of plants of different types determine the water usage, the independent variables should be the numbers of plants. The dependent variable in the function to be minimized should be total water used. Let c represent the number of cucumber plants and t represent the number of lettuce plants.

Step 2 Write a system of inequalities.

Determine the maximum number of each type of plant, given a 30-square-foot constraint and the space requirement for each plant.

Cucumber

$$2.25c \leq 30$$

$$c \leq 13\frac{1}{3}$$

space required for each plant

Divide.

Lettuce

$$1.5t \leq 30$$

$$t \leq 20$$

(continued on the next page)

Go Online You can complete an Extra Example online.



Math History Minute

In the 1960s, **Christine Darden (1942–)** became one of the “human computers” who crunched numbers for engineers at NASA's Langley Research Center. After earning a doctorate degree in mechanical engineering, Darden became one of few female aerospace engineers at NASA Langley. For most of her career, her focus was sonic boom minimization.

Problem Solving Tip

Make a Table You may find it helpful to organize the information in a table before writing the inequalities.

Because it is not possible to plant $13\frac{1}{3}$ cucumber plants, it is more appropriate to limit the number of cucumber plants to 13.

Avoree also wants to have at least 4 of each type of plant, so 4 must be included as minimums in the inequalities. The total number of plants must be at least 16. The total planting area of the plants must be less than or equal to 30 ft².

$$4 \leq c \leq 13$$

$$4 \leq t \leq 20$$

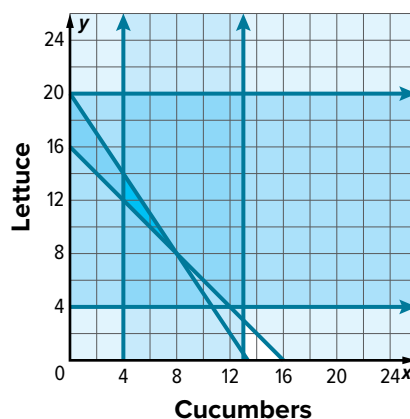
$$c + t \geq 16$$

$$2.25c + 1.5t \leq 30$$

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

The vertices of the feasible region are (4, 12), (4, 14), and (8, 8).



Step 5 Write a linear function to be minimized.

Because Avoree wants to minimize her water usage, the linear function will be the sum of the water usage for each plant.

$$f(c, t) = 25c + 17t$$

Step 6 Evaluate the function at each vertex.

(c, t)	$25c + 17t$	$f(c, t)$
(4, 12)	$25(4) + 17(12)$	304
(4, 14)	$25(4) + 17(14)$	338
(8, 8)	$25(8) + 17(8)$	336

Step 7 Interpret the results.

Avoree should plant 4 cucumber plants and 12 lettuce plants to minimize her water usage.

Check

SOCCER A new soccer team needs to hire players. They need at least 10 defenders and forwards, and they want to minimize the amount they spend on player salaries. Determine the number of forwards f and defenders d they should hire to minimize the cost.

Position	Minimum	Maximum	Salary per Player (\$)
forward f	5	8	120,000
defender d	7	10	100,000

The least amount of money that the team can spend is \$ 1,300,000 by hiring 5 forwards and 7 defenders.

Go Online You can complete an Extra Example online.

Think About It!

Does this solution seem reasonable? Explain.

Yes; sample answer: because cucumber plants use much more water and space than lettuce plants, it makes sense that only the minimum number of cucumber plants should be used.