Solving Systems of Inequalities

Explore Solutions of Systems of Inequalities

Online Activity Use a graph to complete the Explore.

INQUIRY How is a graph used to determine viable solutions of a system of inequalities?

Today's Goal

- Solve systems of linear inequalities in two variables.

Today's Vocabulary

system of inequalities

feasible region

bounded

unbounded

Learn Solving Systems of Inequalities in Two Variables

A **system of inequalities** is a set of two or more inequalities with the same variables. The **feasible region** is the intersection of the graphs. Ordered pairs within the feasible region are viable solutions. The feasible region may be **bounded**, if the graph of the system is a polygonal region, or **unbounded** if it forms a region that is open.

Key Concept • Solving Systems of Inequalities

Step 1 Graph each inequality by graphing the related equation and shading the correct region.

Step 2 Identify the feasible region that is shaded for all of the inequalities. This represents the solution set of the system.

Study Tip

Related Equation A related equation of the inequality $y \leq mx + b$ is $y = mx + b$. The inequalities $y < mx + b$, $y \geq mx + b$, and $y > mx + b$ all share this same related equation.

Go Online

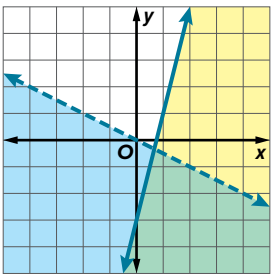
You can watch a video to see how to graph a system of linear inequalities.

Example 1 Unbounded Region

Solve the system of inequalities.

$y \leq 4x - 3$ Inequality 1

$-2y > x$ Inequality 2



Use a solid line to graph the first boundary $y = 4x - 3$. The appropriate half-plane is shaded yellow. Use a dashed line to graph the second boundary $y = -0.5x$. The appropriate half-plane is shaded blue.

The solution of the system is the set of ordered pairs in the intersection of the graphs shaded in green. The feasible region is unbounded.

Study Tip

Boundaries The boundaries of inequalities with symbols $<$ and $>$ are graphed using dashed lines, indicating that the ordered pairs on the boundary are not included in the feasible region.

(continued on the next page)

CHECK

Test the solution by substituting the coordinate of a point in the unbounded region, such as (2, −3), into the system of inequalities. If the point is viable for both inequalities, it is a solution of the system.

$y \leq 4x - 3$	Original inequality	$-2(y) > x$
$-3 \stackrel{?}{\leq} 4(2) - 3$	$x = 2$ and $y = -3$	$-2(-3) \stackrel{?}{>} 2$
$-3 \leq 5$	True	$6 > 2$

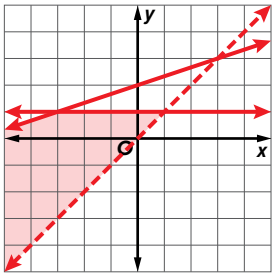
Check

Graph the solution of the system of inequalities.

$y \leq \frac{1}{3}x + 2$

$y > x$

$y \leq 1$



Think About It!

How can you find the coordinates of the vertices of a polygon formed by the system of inequalities?

Sample answer: I can make systems of equations using the related equations of the intersecting boundaries and solve for *x* and *y*.

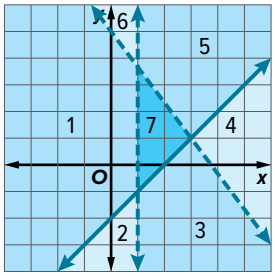
Example 2 Bounded Region

Solve the system of inequalities.

$y < -\frac{4}{3}x + 5$ Inequality 1

$y \geq x - 2$ Inequality 2

$x > 1$ Inequality 3



Use a dashed line to graph the first boundary $y = -\frac{4}{3}x + 5$.

The appropriate shaded area contains regions 1, 2, 3, and 7.

Use a solid line to graph the second boundary $y = x - 2$. The appropriate shaded area contains regions 1, 5, 6, and 7.

Use a dashed line to graph the third boundary $x = 1$. The appropriate shaded area contains regions 3, 4, 5 and 7.

The solution of the system is the set of ordered pairs in the intersection of the graphs, represented by region 7. The feasible region is bounded.

Go Online You can complete an Extra Example online.

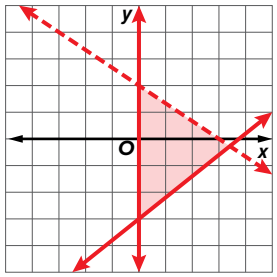
Check

Graph the solution of the system of inequalities.

$y \geq \frac{4}{5}x - 3$

$y < -\frac{2}{3}x + 2$

$x \geq 0$



Example 3 Use Systems of Inequalities

TOURS **A Niagara Falls boat tour company charges $19.50 for adult tickets and $11 for children's tickets. Each boat has a capacity of 600 passengers, including 8 boat crew members. Suppose the company's operating cost for one boat tour is $2750. Write and graph a system of inequalities to represent the situation so the touring company will make a profit on each tour. Then, identify some viable solutions.**

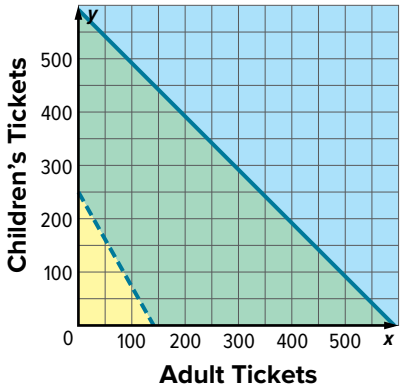
Part A Write the system of inequalities.

Let a represent the number of adult tickets and c represent the number of children's tickets.

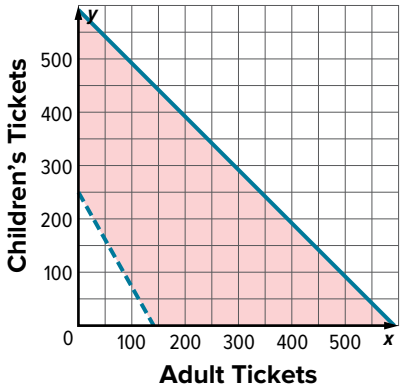
Inequality 1: $a + c + 8 \leq$ 600 Inequality 2: 19.5 $a +$ 11 $c > 2750$

Part B Graph the system of inequalities.

Graph both inequalities.



Identify feasible region.



Part C Identify some viable solutions.

Passengers	Viable	Nonviable
60 adults, 100 children	☐	☒
210 adults, 350 children	☒	☐
415 adults, 200 children	☐	☒
390 adults, 240 children	☐	☒
550 adults, 0 children	☒	☐

Go Online You can complete an Extra Example online.

Your Notes

Talk About It!

Why is it important to label the axes given the context of this problem? Explain.

Sample answer: Because either adult tickets or children's tickets could represent the independent variable, the graph could be made with either variable on the *x*-axis. It is important to label the graph so that viable solutions can be determined correctly.

Study Tip

Consider the Context

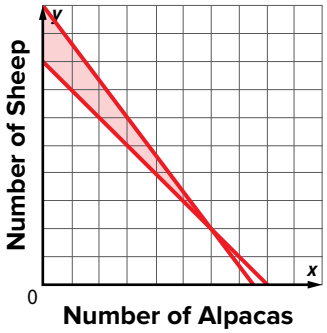
While the feasible region represents the viable solutions, the solution may be limited to only integers or only positive numbers. In this case, the touring company cannot sell a fraction of a ticket. So the solution must be given as whole numbers.

Check

FUNDRAISER The international club raised $1200 to buy livestock for a community in a different part of the world. The club can buy an alpaca for $160 and a sheep for $120. If the club wants to donate at least 8 animals, determine the system of inequalities to represent the situation.

Part A

Graph of the system of inequalities that represents the possible combinations of animals the club can donate.



Part B

Select all of the viable solutions given the constraints of the club's funds. A, B, C

A. 0 alpacas, 10 sheep

B. 1 alpaca, 8 sheep

C. 3 alpacas, 6 sheep

D. 6 alpacas, 3 sheep

E. 8 alpacas, 0 sheep

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

Record your observations here.

See students' observations.

Go Online You can complete an Extra Example online.