Learn Solving Systems of Equations in Two Variables by Substitution

An algebraic method to solve a system of equations is a process called substitution, in which one equation is solved for one variable in terms of the other.

Key Concept • Substitution Method

- Step 1 When necessary, solve at least one equation for one of the variables.
- Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.
- Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 1 Substitution When There Is One Solution

Use substitution to solve the system of equations.

$$8x - 3y = -1$$
 Equation 1
 $x + 2y = -12$ Equation 2

Step 1 Solve one equation for one of the variables.

Because the coefficient of *x* in Equation 2 is 1, solve for *x* in that equation.

$$x + 2y = -12$$
 Equation 2
 $x = \frac{-2y - 12}{}$ Subtract 2y from each side.

Step 2 Substitute the expression. Substitute for x. Then solve for y.

$$8x - 3y = -1$$
 Equation 1

$$8(-2y - 12) - 3y = -1$$

$$x = -2y - 12$$

$$-16y - 96 - 3y = -1$$
 Distributive Property

$$-19y - 96 = -1$$
 Simplify.

$$-19y = 95$$
 Add 96 to each side.

$$y = -5$$
 Divide each side by -19.

Step 3 Substitute to solve.

Substitute the value of *y* into one of the original equations to solve for *x*.

$$x + 2y = -12$$
 Equation 2
 $x + 2(\underline{-5}) = -12$ $y = -5$
 $x = \underline{-2}$ Simplify.

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Today's Goals

- · Solve systems of equations by using the substitution method.
- · Solve systems of equations by using the elimination method.

Today's Vocabulary

substitution elimination



You can watch a video to see how to use algebra tiles to solve a system of equations by using substitution.

Talk About It!

Describe the benefit of solving a system of equations by substitution instead of graphing when the coefficients are not integers.

Sample answer: If the coefficients are not integers, it may be difficult to find the exact solution using a graph. Since an exact solution can be calculated using substitution, it would be a better method.



Think About It!

What can you conclude about the slopes and y-intercepts of the equations when a system of equations has no solution? when a system of equations has infinitely many solutions?

Sample answer: When a system of equations has no solution, the slopes of the equations are the same, but the *y*-intercepts are different. When a system of equations has infinitely many solutions, the slopes and y-intercepts of the equations are the same.



Think About It!

Explain what approximations were made while solving this problem and how they affect the solution.

Sample answer: The value of y is rounded in Step 3 since it would not be practical to measure to the exact value of y. Because x is calculated based on the value of y, it is also an approximation. When two rounded values are used to check the solution, the equations are only approximately equal.

Check

Use substitution to solve the system of equations.

$$-5x + y = -3$$

$$3x - 8y = 24$$
 (0, -3)

Example 2 Substitution When There Is Not Exactly One Solution

Use substitution to solve the system of equations.

$$-5x + 2.5y = -15$$
 Equation 1
 $y = 2x - 11$ Equation 2

Equation 2 is already solved for y, so substitute 2x - 11 for y in Equation 1.

$$-5x + 2.5y = -15$$
 Equation 1
 $-5x + 2.5(\frac{2x - 11}{2}) = -15$ $y = 2x - 11$
 $-5x + \frac{5}{2}x - \frac{27.5}{2} = -15$ Distributive Property
 $\frac{-27.5}{2} = -15$ False

This system has $\frac{\text{no solution}}{\text{because } -27.5} = -15$ is not true.

Example 3 Apply the Substitution Method

CHEMISTRY Ms. Washington is preparing a hydrochloric acid (HCI) solution. She will need 300 milliliters of a 5% HCl solution for her class to use during a lab. If she has a 3.5% HCl solution and a 7% HCl solution, how much of each solution should she use in order to make the solution needed?

Step 1 Write two equations in two variables.

Let x be the amount of 3.5% solution and y be the amount of 7% solution.

$$x + y = 300$$
 Equation 1
0.035 $x + 0.07y = 0.05(300)$ Equation 2

Step 2 Solve one equation for one of the variables.

$$x + y = 300$$
 Equation 1
 $x = \underline{y} + 300$ Subtract y from each side.

Step 3 Substitute the resulting expression and solve.

$$0.035x + 0.07y = 15$$
 Equation 2
 $0.035 \frac{(-y + 300)}{0.035} + 0.07y = 15$ $x = -y + 300$
 $-0.035y + 10.5 + 0.07y = 15$ Distributive Property
 $0.035y = 4.5$ Simplify.
 $y \approx 128.57$ Divide each side by 0.035.

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(continued on the next page)

$$x + y = 300$$
 Equation 1
 $x + 128.57 \approx 300$ $y \approx 128.57$
 $x \approx 171.43$ Simplify.

The solution of the system is (171.43, 128.57). Ms. Washington should use 171.43 mL of the 3.5% solution and 128.57 mL of the 7% solution.

Learn Solving Systems of Equations in Two Variables by Elimination

Systems of equations may be solved algebraically using elimination, which is the process of using addition or subtraction to eliminate one variable and solve a system of equations.

Key Concept • Elimination Method

- Step 1 Multiply one or both of the equations by a number to result in two equations that contain opposite or equal terms.
- **Step 2** Add or subtract the equations, eliminating one variable. Then solve the equation.
- Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 4 Elimination When There Is One Solution

Use elimination to solve the system of equations.

$$-2x - 9y = -25$$
 Equation 1
 $-4x - 9y = -23$ Equation 2

Step 1 Multiply the equation.

Multiply Equation 2 by -1 to get opposite terms -9y and 9y.

$$-4x - 9y = -23$$
 Multiply by -1. $4x + 9y = 23$

Step 2 Add the equations.

Add the equations to eliminate the y-term and solve for x.

$$-2x - 9y = -25$$
 Equation 1

$$(+) 4x + 9y = 23$$
 Equation 2 × (-1)

$$2x = -2$$
 Add the equations.

$$x = -1$$
 Divide each side by 2.

Step 3 Substitute and solve.

$$-4x - 9y = -23$$

$$-4(-1) - 9y = -23$$

$$-9y = -27$$

$$y = 3$$
Substitute -1 for x in Equation 2.
$$x = -1$$
Simplify.
$$y = 3$$
Divide each side by -9.

The solution of the system is (-1, 3).

Think About It!

When using elimination, when should you add the equations, and when should you subtract the equations?

Sample answer: You would add the equations when they have terms with opposite coefficients. You would subtract the equations when they have terms with the same coefficients.

Think About It!

Describe the benefit of using elimination instead of substitution of this problem.

Sample answer: Solving for *x* or y in either of the equations of the system would result in noninteger coefficients. That would make using substitution much more difficult.



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Example 5 Multiply Both Equations Before Using Elimination

Use elimination to solve the system of equations.

$$2x + 5y = 1$$
 Equation 1
 $3x - 4y = -10$ Equation 2

Step 1 Multiply one or both equations.

Multiply Equation 1 by 3 and Equation 2 by 2.

$$2x + 5y = 1$$
 Original equations $3x - 4y = -10$
 $3(2x + 5y) = 3(1)$ Multiply. $2(3x - 4y) = 2(-10)$
 $6x + 15y = 3$ Distribute. $6x - 8y = -20$

Step 2 Eliminate one variable and solve.

In order to eliminate the *x*-terms, subtract the equations. Then, solve for *y*.

$$6x + 15y = 3$$
 Equation 1 × 3

$$(-) 6x - 8y = -20$$
 Equation 2 × 2

$$23y = 23$$
 Subtract the equations.

$$y = 1$$
 Divide each side by 23.

Step 3 Substitute and solve.

Substitute y = 1 in either of the original equations and solve for x.

$$2x + 5y = 1$$
 Equation 1

$$2x + 5(1) = 1$$
 $y = 1$

$$2x + 5 = 1$$
 Multiply.

$$x = -2$$
 Solve for x.

The solution of the system is (-2, 1).

Example 6 Elimination Where There is Not Exactly One Solution

Use elimination to solve the system of equations.

$$18x + 21y = 14$$
 Equation 1

 $6x + 7y = 2$
 Equation 2

Steps 1 and 2 Multiply one or both equations and add them.

Multiply Equation 2 by -3. Then add the equations.

$$18x + 21y = 14$$

 $6x + 7y = 2$
Multiply by -3.
$$18x + 21y = 14$$

$$(+) -18x - 21x = -6$$

$$0 \neq 8$$

Because $0 \neq 8$, this system has no solution.

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Think About It!

Describe the graph of this system of equations.

Sample answer: The graph is two parallel lines.

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to practice what you've learned about solving systems of equations in the Put It All Together over Lessons 2-4 and 2-5.