Solving Systems of Equations Graphically
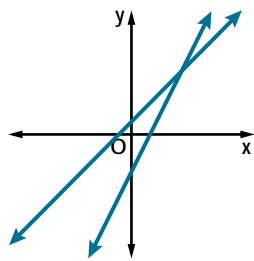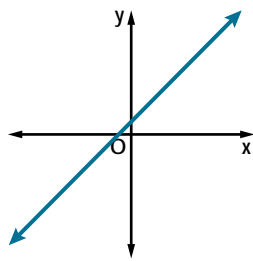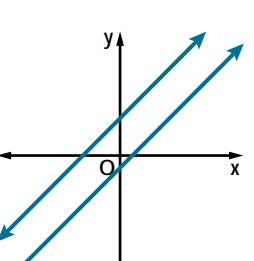
Explore Solutions of Systems of Equations

Online Activity Use graphing technology to complete the Explore.

> ☒
> **INQUIRY** How is the solution of a system of equations represented on a graph?

Learn Solving Systems of Equations in Two Variables by Graphing

A **system of equations** is a set of two or more equations with the same variables. One method for solving a system of equations is to graph the related function for each equation on the same coordinate plane. The point of intersection of the two graphs represents the solution.

Types of Graphs		
The lines intersect at one point. The equations have different slopes.	The lines are identical. The equations have the same slope and *y*-intercept.	The lines are parallel. The equations have the same slope and different *y*-intercepts.
Solutions		
one solution	infinitely many solutions	no solutions
Classifications		
The system is **consistent** because there is at least one solution. It is **independent** because it has exactly one solution.	The system is **consistent** and **dependent** because there are infinitely many solutions.	The system is **inconsistent** because there is no solution.

Today's Goal
- Solve systems of linear equations by graphing.

Today's Vocabulary
system of equations
consistent
inconsistent
independent
dependent

🗩 Talk About It!
Explain why the intersection of the two graphs is the solution of the system of equations.

Sample answer: The graph of a line represents every point that is a solution for the equation of that line. So when the graphs of two equations intersect, the point of intersection lies on both lines, meaning that it is a solution of both equations.

Example 1 Classify Systems of Equations

Determine the number of solutions the system has. Then state whether the system of equations is *consistent* or *inconsistent* and whether it is *independent* or *dependent*.

Solve each equation for y.

$2y = 6x - 14 \quad \rightarrow \quad y = 3x - 7$

$3x - y = 7 \quad\quad \rightarrow \quad y = \underline{\ \textbf{3x} - \textbf{7}\ }$

The equations have the same slope and y-intercept. Thus, both equations represent the same line and the system has __infinitely many__ __solutions__. The system is __consistent__ and __dependent__.

Check

Determine the number of solutions and classify the system of equations. __one solution; consistent and independent__

$3x - 2y = -7$

$4y = 9 - 6x$

Example 2 Solve a System of Equations by Graphing

Solve the system of equations.

$5x - y = 3$

$-x + y = 5$

Solve each equation for y. The equations have different slopes, so there is one solution. Graph the system.

$5x - y = 7 \quad \rightarrow \quad \underline{\ y = \textbf{5x} - \textbf{3}\ }$

$-x + y = 5 \quad \rightarrow \quad \underline{\ y = \textbf{x} + \textbf{5}\ }$

The lines appear to intersect at one point, ($\underline{\textbf{2}}$, $\underline{\textbf{7}}$).

CHECK Substitute the coordinates into each original equation.

	Original equation	
$5x - y = 3$		$-x + y = 5$
$5(2) - 7 = 3$	$x = 2$ and $y = 7$	$-(2) + 7 = 5$
$3 = 3$	True	$5 = 5$

The solution is (2, 7).

Check

Solve the system of equations by graphing.

$2y + 14x = -6$

$8x - 4y = -24$

The solution is ($\underline{\ -\textbf{1}, \textbf{4}\ }$).
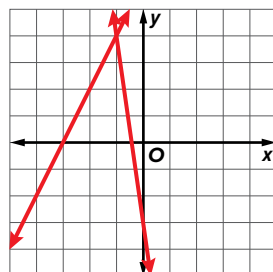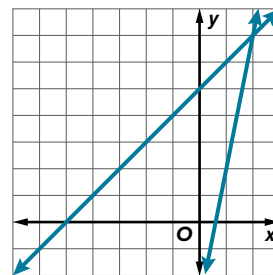
Example 3 Solve a System of Equations

Solve the system of equations.

7x + 2y = 16

−21x − 6y = 24

Solve each equation for y to determine the number of solutions the system has.

$7x + 2y = 16 \quad \rightarrow \quad y = \underline{-3.5}x + \underline{8}$

$-21x - 6y = 24 \quad \rightarrow \quad y = \underline{-3.5}x + \underline{-4}$

The equations have the ___**same**___

slope and ___**different**___ y-intercepts. So,

these equations represent ___**parallel**___

lines, and there is **no solution**.

You can graph each equation on the same

grid to confirm that they do not intersect.

🌐 Example 4 Write and Solve a System of Equations by Graphing

CARS **Suppose an electric car costs $29,000 to purchase and $0.036 per mile to drive, and a gasoline-powered car costs $19,000 to purchase and $0.08 per mile to drive. Estimate after how many miles of driving the total cost of each car will be the same.**

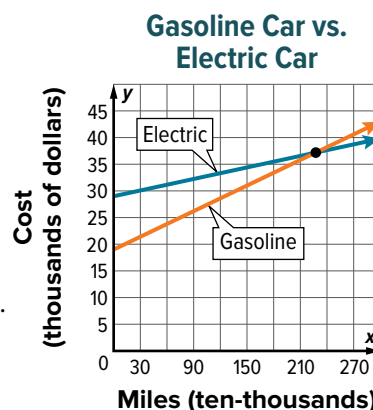**Part A Write equations for the total cost of owning each type of car.**

Let y = the total cost of owning the car and x = the number of miles driven.

So, the equation is $y = \underline{0.036}x + \underline{29,000}$ for the electric car and

$y = \underline{0.08}x + \underline{19,000}$ for the gasoline car.

Part B Examine the graph to estimate the number of miles you would have to drive before the cost of owning each type of car would be same.

The graphs appear to intersect at

approximately ($\underline{225,000}$, $\underline{37,500}$).

This means that after driving

about $\underline{225,000}$ miles, the cost of

owning each car will be the same.

Gasoline Car vs. Electric Car

(continued on the next page)

Study Tip

Parallel Lines Graphs of lines with the same slope and different intercepts are, by definition, parallel.

💭 Think About It!

What would the graph of a system with infinitely many solutions look like? Explain your reasoning.

Sample answer: For a system to have infinitely many solutions, both equations must represent the same line. Therefore, the graph of the system would be a single line.

💭 Think About It!

Explain what the two equations represent in the context of the situation.

Sample answer: Each equation represents the cost of owning a car after driving x miles. The equation represents the sum of the initial cost of the car and the cost of driving each mile based on the price of its energy source.

CHECK Substitute the coordinates into each original equation.

$$0.036x + 29{,}000 = y \qquad\qquad 0.08x + 19{,}000 = y$$
$$0.036(225{,}000) + 29{,}000 \stackrel{?}{=} 37{,}500 \qquad 0.08(225{,}000) + 19{,}000 \stackrel{?}{=} 37{,}500$$
$$37{,}100 \approx 37{,}500 \qquad\qquad 37{,}000 \approx 37{,}500$$

The estimated number of miles makes both equations approximately true. So, our estimate is reasonable.

Example 5 Solve a System by Using Technology

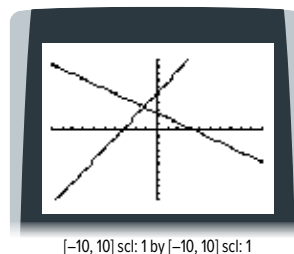**Use a graphing calculator to solve the system of equations.**

Step 1 Solve for y.

$$3.5y - 5.6x = 18.2 \quad \rightarrow \quad y = \underline{1.6}\,x + \underline{5.2}$$
$$-0.7x - y = -2.4 \quad \rightarrow \quad y = \underline{-0.7}x + \underline{2.4}$$

Step 2 Graph the system.

Enter the equations in the **Y=** list and graph in the standard viewing window.

Step 3 Find the intersection.

Use the **intersect** feature from the **CALC** menu to find the coordinates of the point of intersection. When prompted, select each line. Press **enter** to see the intersection.

[−10, 10] scl: 1 by [−10, 10] scl: 1

The solution is approximately (<u>−1.22, 3.25</u>).

Example 6 Solve a Linear Equation by Using a System

Use a graphing calculator to solve $4.5x - 3.9 = 6.5 - 2x$ by using a system of equations.
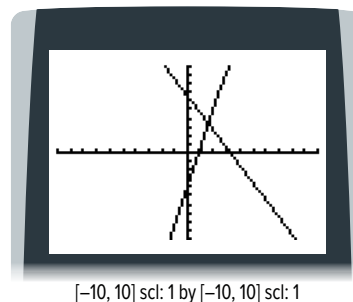
Step 1 Write a system.

Set each side of $4.5x - 3.9 = 6.5 - 2x$ equal to y to create a system of equations.

$$y = \underline{4.5x - 3.9}$$
$$y = \underline{6.5 - 2x}$$

Step 2 Graph the system.

Enter the equations in the **Y=** list and graph in the standard viewing window.

Step 3 Find the intersection.

The solution is the x-coordinate of the intersection, which is <u>1.6</u>.

[−10, 10] scl: 1 by [−10, 10] scl: 1

🏹 **Go Online** You can complete an Extra Example online.