Solving Linear Equations and Inequalities

Learn Solving Linear Equations

An **equation** is a mathematical sentence stating that two mathematical expressions are equal. The **solution** of an equation is a value that makes the equation true. To solve equations, use the properties of equality to isolate the variable on one side.

Key Concept • Properties of Equality

Property of Equality	Symbols
Addition Property of Equality	For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For any real numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $ac = bc$.
Division Property of Equality	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $\frac{a}{b} = \frac{b}{c}$.

Example 1 Solve a Linear Equation

Solve $\frac{1}{3}(2x - 57) + \frac{1}{3}(6 - x) = -4$.

$\frac{1}{3}(2x - 57) + \frac{1}{3}(6 - x) = -4$	Original equation
$\frac{2}{3}x - \underline{\textbf{19}} + \underline{\textbf{2}} - \frac{1}{3}x = -4$	Distributive Property
$\underline{\frac{1}{3}x} - \underline{\textbf{17}} = -4$	Combine like terms.
$\frac{1}{3}x = \underline{\textbf{13}}$	Add 17 to each side and simplify.
$x = \underline{\textbf{39}}$	Multiply each side by 3 and simplify.

🌐 Example 2 Write and Solve an Equation

SPACE The diameter of Earth is 828 kilometers less than twice the diameter of Mars. If Earth has a diameter of 12,756 kilometers, what is the diameter of Mars?

Part A Write an equation that represents the situation.

Words	The diameter of Earth	is	823 less than twice the diameter of Mars.
Variable	Let $m = $ **the diameter of Mars**.		
Equation	**12,756**	$=$	$2m - $ **828**

🌐 **Go Online** You can complete an Extra Example online.

*(**continued on the next page**)*

Study Tip

Justifications The properties of equality can be used as justifications. However, in most future solutions, the justifications for steps will read as "Subtract c from each side," or "Divide each side by c."

🌐 **Go Online**
You can watch a video to see how to solve equations in one variable using a graphing calculator.

Part B Solve the equation.

$$12{,}756 = 2m - 288 \qquad \text{Original equation}$$

$$12{,}756 + \underline{\textbf{828}} = 2m - 288 + 888 \qquad \text{Add 828 to each side.}$$

$$\underline{\textbf{13,584}} = 2m \qquad \text{Simplify.}$$

$$\frac{13{,}584}{2} = \frac{2m}{2} \qquad \text{Divide each side by 2.}$$

$$\underline{\textbf{6792}} = m \qquad \text{Simplify.}$$

The diameter of Mars is $\underline{\textbf{6792}}$ kilometers. This is a reasonable solution because 12,756 is a little less than 6792 • 2 = 13,584, as indicated in the problem.

Check

BASKETBALL In 1962, Wilt Chamberlain set the record for the most points scored in a single NBA game. He scored 28 points from free throws and made *x* field goals, worth two points each. If Wilt Chamberlain scored 100 points, how many field goals did he make? Which equation represents the number of field goals that Chamberlain scored? $\underline{\textbf{A}}$

A. $100 = 28 + 2x$ **B.** $100 = 28x + 2$ **C.** $28 = 2x$ **D.** $100 = 2x$

How many field goals did Chamberlain score? $\underline{\textbf{36}}$ field goals

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Example 3 Solve for a Variable

GEOMETRY The formula for the perimeter of a parallelogram is $P = 2a + 2b$ where *a* and *b* represent the measures of the bases. Solve the equation for *b*.

$$P = 2a + 2b \qquad \text{Original equation}$$

$$P - \underline{\textbf{2a}} = 2a + 2b - \underline{\textbf{2a}} \qquad \text{Subtract } 2a \text{ from each side.}$$

$$P - 2a = \underline{\textbf{2b}} \qquad \text{Simplify.}$$

$$\frac{P}{2} - \frac{2a}{2} = \frac{2b}{2} \qquad \text{Divide each side by 2.}$$

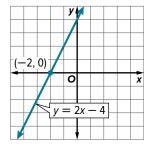$$\frac{P}{2} - \underline{\textbf{a}} = b \qquad \text{Simplify.}$$

Check

GEOMETRY The formula for the area *A* of a trapezoid is solved for *h*. Fill in the missing justification.

$$A = \tfrac{1}{2}h(a + b) \qquad \text{Original equation}$$

$$2A = 2 \cdot \tfrac{1}{2}h(a + b) \qquad \text{Multiplication Property of Equality}$$

$$2A = h(a + b) \qquad \text{Simplify.}$$

$$\frac{2A}{(a + b)} = \frac{h(a + b)}{(a + b)} \qquad \underline{\textbf{Division Property of Equality}}$$

$$\frac{2A}{(a + b)} = h \qquad \text{Simplify.}$$

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Think About It!
What does it mean to solve for a variable?

Sample answer: Isolate the variable on one side of the equation.

Learn Solving Linear Equations by Graphing

The solution of an equation is called a **root**. You can find the root of an equation by examining the graph of its related function $f(x)$. A related function is found by solving the equation for 0 and then replacing 0 with $f(x)$. A related function for $2x + 13 = 9$ is $f(x) = 2x + 4$ or $y = 2x + 4$.

Values of x for which $f(x) = 0$ are called **zeros** of the function f. The zero of a function is the x-intercept of its graph. The solution and root of a linear equation are the same as the zero and x-intercept of its related function.

Example 4 Solve a Linear Equation by Graphing
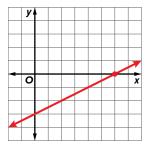
Step 1 Find a related function.

Rewrite the equation with 0 on the right side.

$$\frac{1}{2}x - 11 = -8 \qquad \text{Original equation.}$$

$$\frac{1}{2}x - 11 + 8 = -8 + 8 \qquad \text{Add 8 to each side.}$$

$$\underline{\frac{1}{2}x - 3} = 0 \qquad \text{Simplify.}$$

Replacing 0 with $f(x)$ gives the related function, $\underline{f(x) = \frac{1}{2}x - 3}$.

Step 2 Graph the related function.

Since the graph of $f(x) = \frac{1}{2}x - 3$ intersects the x-axis at **6** , the solution of the equation is **6**.
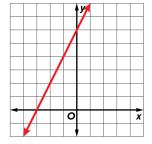

Check

Graph the related function of $2x - 5 = -11$. Use the graph to solve the equation.

The related function is $\underline{y = 2x + 6}$.

The solution is $\underline{-3}$.

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Study Tip

Assumptions
Assuming that the rate at which Paul Crake climbed the stairs was constant allows us to represent the situation with a linear equation. While the rate at which he climbed likely varied throughout the race, using constant rates allows for reasonable graphs and solutions.

🌐 **Example 5** Estimate Solutions by Graphing

TOWER RACE **The Empire State Building Run-Up is a race in which athletes run up the building's 1576 stairs. In 2003, Paul Crake set the record for the fastest time, running up an average of about 165 stairs per minute. The function $c = 1576 - 165m$ represents the number of steps Crake had left to climb c after m minutes. Find the zero of the function and interpret its meaning in the context of the situation.**

Step 1 Graph the function.

Step 2 Estimate the zero.

The graph appears to intersect the

x-axis at about __9.5__. This means that

Paul Crake finished the race in about

9.5 minutes, or __9__ minutes and

__30__ seconds.

Tower Race

Step 3 Solve algebraically.

Write the function as an equation set equal to 0, and solve algebraically to check your solution.

$$c = 1576 - 165m \qquad \text{Original equation}$$
$$0 = 1576 - 165m \qquad \text{Replace } c \text{ with 0.}$$
$$\underline{165m} = \underline{1576} \qquad \text{Add } 165m \text{ to each side.}$$
$$m \approx \underline{9.55} \qquad \text{Divide each side by 165.}$$
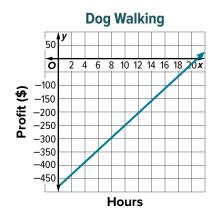
The solution is about __9.55__. So, Paul Crake completed the Empire State Building Run-Up in about 9.55 minutes, or __9__ minutes and __33__ seconds. This is close to the estimated time of 9.5 minutes.

Check

DOG WALKING Bethany spends $480 on supplies to start a dog walking service for which she plans to charge $23 per hour. The function $y = 23x - 480$ represents Bethany's profit after x hours of dog walking.

Part A The graph appears to intersect the x-axis at about __20.9__.

Part B Solve algebraically to verify your estimate. Round to the nearest hundredth. __20.87__

Dog Walking

🧭 **Go Online** You can complete an Extra Example online.

Explore Comparing Linear Equations and Inequalities

Online Activity Use a comparison to complete the Explore.

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INQUIRY How do the solution methods and the solutions of linear equations and inequalities in one variable compare?
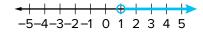
Learn Solving Linear Inequalities

An **inequality** is an open sentence that contains the symbol $<$, $\leq$, $>$, $\geq$. Properties of inequalities allow you to perform operations on each side of an inequality without changing the truth of the inequality.

Key Concept • Addition and Subtraction Properties of Inequality

Symbols	For any real numbers, a, b, and c:	
	If $a > b$, then $a + c > b + c$. If $a > b$, then $a - c > b - c$.	
	If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.	
Examples	$2 > 0$	$9 > 6$
	$2 + 1 > 0 + 1$	$9 - 4 > 6 - 4$
	$3 > 1$	$5 > 2$

Key Concept • Multiplication and Division Properties of Inequality

Symbols	For any real numbers, a, b, and c:	
	where c is positive:	
	If $a > b$, then $ac > bc$. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$.	
	If $a < b$, then $ac < bc$. If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	
	where c is negative:	
	If $a > b$, then $ac < bc$. If $a > b$, then $\frac{a}{c} < \frac{b}{c}$.	
	If $a < b$, then $ac > bc$. If $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	
Examples	$8 > -2$	$-4 < -2$
	$8(3) > -2(3)$	$-4(-8) < -2(-8)$
	$24 > -6$	$32 > 16$

The solution sets of inequalities can be expressed by using set-builder notation. For example, $\{x \mid x > 1\}$ represents the set of all numbers x such that x is greater than 1. The solution sets can also be graphed on number lines. Circles and dots are used to indicate whether an endpoint is included in the solution. The circle at 1 means that this point is *not* included in the solution set.

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You may want to complete the Concept Check to check your understanding.

Think About It!
What does the arrow on the graph of a solution set represent?

Sample answer: The arrow indicates that all of the values in that direction are included in the solution set.

Study Tip

Reversing the Inequality Symbol Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality. Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a negative number requires that the order of the inequality be reversed. In Example 6, $\geq$ was replaced with $\leq$.
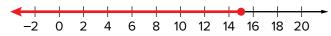
Watch Out!

Reading Math Be sure to always read problems carefully. The term *at least* is used here, and can be confusing since it actually means *greater than or equal to*, and is represented by $\geq$. In this instance, Jake should intake at least 1300 mg, which means he must intake an amount greater than or equal to 1300 mg.

Example 6 Solve a Linear Inequality

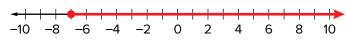**Solve $-5.6n + 12.9 \geq -71.1$. Graph the solution set on a number line.**

$-5.6n + 12.9 \geq -71.1$	Original inequality.
$-5.6n + 12.9 \underline{-\ 12.9} \geq -71.1 \underline{-\ 12.9}$	Subtract 12.9 from each side.
$-5.6n \geq -84$	Simplify.
$\dfrac{-5.6n}{-5.6} \leq \dfrac{-84}{-5.6}$	Divide each side by -5.6, reversing the inequality symbol.
$n \leq \underline{\ 15\ }$	Simplify.

The solution set is $\{n \mid n \leq \underline{\ 15\ }\}$. Graph the solution set.

Check

What is the solution of $-p - 3 \geq -4(p + 6)$? **$p \geq -7$**
Graph the solution set.

🌐 Example 7 Write and Solve an Inequality

NUTRITION The recommended daily intake of calcium for teens is 1300 mg. Jake gets 237 mg of calcium from a multivitamin he takes each morning and 302 mg from each glass of skim milk that he drinks. How many glasses of milk would Jake need to drink to meet the recommendation?

Step 1 Write an inequality to represent the situation.

Let $g =$ the number of __**glasses of milk**__ Jake needs.

__**237**__ $+ 302g$ __**$\geq$**__ 1300

Step 2 Solve the inequality.

$237 + 302g \geq 1300$	Original inequality
$302g \geq \underline{\textbf{1063}}$	Subtract 237 from each side.
$g \geq \underline{\textbf{3.52}}$	Divide each side by 302.

Step 3 Interpret the solution in the context of the situation.

Jake will need to drink __**slightly more than 3.5**__ glasses of milk to intake at least the recommended daily amount of calcium. This is a viable solution because Jake can pour part of a full glass of milk.

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