Graphing Linear Functions and Inequalities

Learn Graphing Linear Functions

The graph of a function represents all ordered pairs that are true for the function. You can use various methods to graph a linear function.

Example 1 Graph by Using a Table

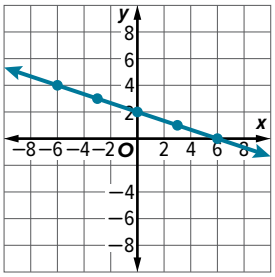
Graph $x + 3y - 6 = 0$ by using a table.

Solve the equation for y. Then make a table and complete the graph.

$x + 3y - 6 = 0$ Original function

$y =$ $-\frac{1}{3}x + 2$ Solve.

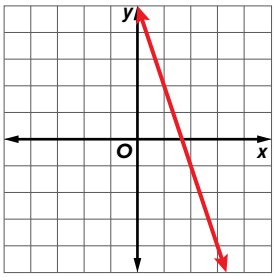
x	$-\frac{1}{3}x + 2$	y
-6	$-\frac{1}{3}(-6) + 2$	4
-3	$-\frac{1}{3}$ (−3) $+ 2$	3
0	$-\frac{1}{3}(0) + 2$	2
3	$-\frac{1}{3}(3) + 2$	1
6	$-\frac{1}{3}$ (6) $+ 2$	0



Check

Graph $6x + 2y = 10$ by using a table.

x	y
−1	8
0	5
1	2
3	−4



Example 2 Graph by Using Intercepts

Graph $3x - 2y = -12$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$3x - 2y = -12$ Original function

$3x - 2($ 0 $) = -12$ Replace with 0.

$x =$ −4 Simplify.

To find the y-intercept, let $x = 0$.

$3x - 2y = -12$

$3($ 0 $) - 2y = -12$

$y =$ 6

(*continued on the next page*)

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Today's Goals

- Graph linear functions.
- Graph linear inequalities in two variables.

Today's Vocabulary

linear inequality

boundary

closed half-plane

open half-plane

constraint

Go Online
You can watch a video to see how to graph linear functions.

Study Tip

Recall that **slope** is the ratio of the change in the y-coordinates (rise) to the corresponding change in the x-coordinates (run) as you move from one point to another along a line.

Think About It!

Explain why −6, −3, 0, 3, and 6 were selected for the x-values in the table.

Sample answer: Because x is multiplied by $-\frac{1}{3}$, using x-values that are multiples of 3 will result in integers, which are easier to plot on the coordinate plane.

Your Notes

Think About It!
How can you check that the graph is correct?

Sample answer: Find another point on the line, such as (−2, 3) and substitute the values of *x* and *y* into the function.

$3(-2) - 2(3) \stackrel{?}{=} -12$

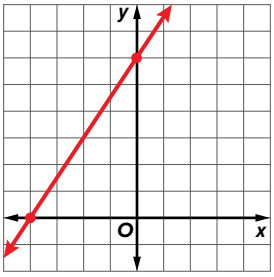
$-6 - 6 \stackrel{?}{=} -12$

$-12 = -12$ True

The *x*-intercept is −4, and the *y*-intercept is 6. This means that the graph passes through (−4, 0) and (0, 6).

Plot the two intercepts.

Draw a line through the points.

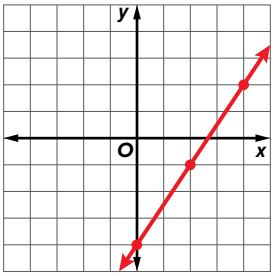


Example 3 Graph by Using the Slope and *y*-intercept

Graph $y = \frac{3}{2}x - 4$ by using *m* and *b*.

Follow these steps.

- Begin by identifying the slope *m* and *y*-intercept *b* of the function.
  $m = \frac{3}{2}$ $b = -4$
- Use the value of *b* to plot the *y*-intercept (0, −4).
- Use the slope of the line $m = \frac{3}{2}$ to plot more points. From the *y*-intercept, move up 3 units and right 2 units. Plot a point at (2, −1).
- From the point (2, −1), move up 3 units and right 2 units. Plot a point at (4, 2).
- Draw a line through the points.



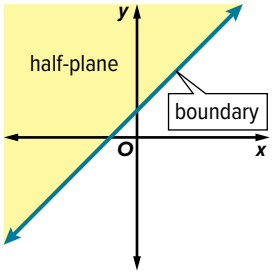
Explore Shading Graphs of Linear Inequalities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use a point to test the graph of an inequality?

Learn Graphing Linear Inequalities in Two Variables

The graph of a **linear inequality** is a half-plane with a boundary that is a straight line. The half-plane is shaded to indicate that all points contained in the region are solutions of the inequality. A **boundary** is a line or curve that separates the coordinate plane into two half-planes.



Go Online
You can watch a video to see how to graph a linear inequality in two variables.

Go Online You can complete an Extra Example online.

The boundary is solid when the inequality contains ≤ or ≥ to indicate that the points on the boundary are included in the solution, creating a **closed half-plane**. The boundary is dashed when the inequality contains < or > because the points on the boundary do not satisfy the inequality. This results in an **open half-plane**.

A **constraint** is a condition that a solution must satisfy. Each solution of the inequality represents a viable, or possible, option that satisfies the constraint.

Example 4 Graph an Inequality with an Open Half-Plane

Graph $12 - 4y > x$.

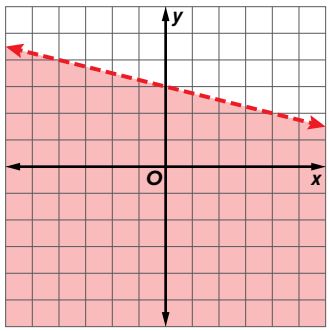
Step 1 Graph the boundary.

$12 - 4y > x$ — Original inequality

$\underline{-4y} > x - 12$ — Subtract 12 from each side.

$y < \underline{-\frac{1}{4}}x + \underline{3}$ — Divide each side by −4, and reverse the inequality symbol.

The boundary of the graph is $y \underline{=} -\frac{1}{4}x + 3$. Because the inequality symbol is >, the boundary is dashed.



Step 2 Use a test point and shade.

Test (0, 0).

$12 - 4y > x$ — Original inequality

$12 - 4(0) \stackrel{?}{>} 0$ — Substitute.

$12 > 0$ — True

Because (0, 0) is a solution of the inequality, shade the half-plane that contains the test point.

Check: Check by selecting another point in the shaded region to test.

Example 5 Graph an Inequality with a Closed Half-Plane

Graph $9 + 3y \leq 8x$.

Step 1 Graph the boundary.

Solve for y in terms of x and graph the related function.

$9 + 3y \leq 8x$ — Original inequality

$3y \leq 8x \underline{-9}$ — Subtract 9 from each side.

$y \leq \underline{\frac{8}{3}}x - \underline{3}$ — Divide each side by 3.

The related equation of $y \leq \frac{8}{3}x - 3$ is $y = \frac{8}{3}x - 3$, and the boundary is solid.

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Study Tip

Above or Below

Usually the shaded half-plane of a linear inequality is said to be *above* or *below* the line of the related equation. However, if the equation of the boundary is $x = c$ for some constant c, then the function is a vertical line. In this case, the shading is considered to be *to the left* or *to the right* of the boundary.

Talk About It!

Can a linear inequality ever be a function? Explain your reasoning.

No; sample answer: for any value of x, there are infinitely many values of y in the solution set of a linear inequality. Therefore, a linear inequality cannot be a function.

Think About It!

Why should you not test a point that is on the boundary?

Sample answer: Testing a point on the boundary does not indicate whether you have properly shaded the solution set. It will only indicate whether the boundary should be included.

Step 2 Use a test point and shade.

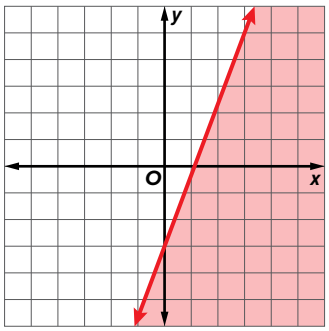
Select a test point, such as (0, 0).

$9 + 3y \leq 8x$ Original inequality

$\underline{9} + 3(\underline{0}) \stackrel{?}{\leq} 8(\underline{0})$ $(x, y) = (0, 0)$.

$9 \not\leq 0$ False

Shade the side of the graph that does not contain the test point.



Apply Example 6 Linear Inequalities

GRADES **Malik's algebra teacher determines semester grades by finding the sum of 70% of a student's test grade average and 30% of a student's homework grade average. If Malik wants a semester grade of 90% or better, write and graph the inequality that represents the constraints for Malik's test grade *x* and homework grade *y*.**

1 What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: Use the description to write the inequality. Find points on the boundary and use a test point to create the graph.

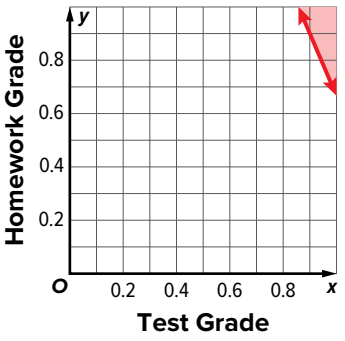
2 How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: Write and graph an inequality to represent the constraints on Malik's grades. How do the test and homework grades relate to the semester grade?

3 What is your solution?

Use your strategy to solve the problem. What inequality represents the constraints for Malik's test and homework grades? Use the grid to graph the inequality.

$0.7x + 0.3y \geq 0.9$



Which of these are viable solutions for Malik's test and homework grades?

- [x] 88% test, 100% homework
- [ ] 95% test, 70% homework
- [x] 90% test, 90% homework
- [x] 95% test, 80% homework
- [ ] 90% test, 80% homework
- [x] 100% test, 70% homework

4 How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution. Sample answer: I can select a point in the shaded region, such as (0.95, 0.8) and test it in the inequality.

Think About It!
Is (3, 5) a solution of the inequality? Explain.

Yes; sample answer: (3, 5) is on the solid boundary, so it is included in the solution.

Go Online
You can watch a video to see how to graph an inequality using a graphing calculator.

Go Online
You can complete an Extra Example online.