Sketching Graphs and Comparing Functions

Explore Using Technology to Examine Key Features of Graphs

Online Activity Use graphing technology to complete the Explore.

INQUIRY What can key features of a function tell you about its graph?

Learn Sketching Graphs of Functions

You can use key features of a function to sketch its graph.

Key Concept • Using Key Features

Key Feature	What it tells you about the graph
Domain	the values of x for which $f(x)$ is defined
Range	the values that $f(x)$ take as x varies
Intercepts	where the graph crosses the x- or y-axes
Symmetry	where one side of the graph is a reflection or rotation of the other side
End Behavior	what the graph is doing at the right and left sides as x approaches infinity or negative infinity
Extrema	high or low points where the graph changes from increasing to decreasing or vice versa
Increasing/ Decreasing	where the graph is going up or down as x increases
Positive/Negative	where the graph is above or below the x-axis

Example 1 Sketch a Linear Function

Use the key features of the function to sketch its graph.

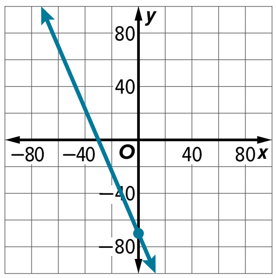
y-intercept: $(0, -70)$

Linearity: linear

Positive: for values of x such that $x < -30$

Decreasing: for all values of x

End Behavior: As $x \to \infty$, $f(x) \to -\infty$.
As $x \to -\infty$, $f(x) \to \infty$.



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Today's Goals

- Sketch graphs of functions by using key features.

Study Tip

Scales and Axes Before you sketch a function, consider the scales or axes that best fit the situation. You want to capture as much information as possible, so you want the scales to be big enough to easily see the extrema and x- and y intercepts, but not so big that you cannot determine the values.

Talk About It!

Given the y-intercept and for what values of x the function is positive, what other information do you need to sketch a linear function? Explain your reasoning.

Sample answer: You do not need any other information. If you know the y-intercept and where $f(x)$ is positive, then you know where the function crosses the x- and y-axis. If the x- and y- intercepts are the same, then you can sketch but not accurately graph the function; if they are different, then you can draw a line through the two points to accurately graph the function.

Your Notes

Study Tip

Assumptions When sketching the function using the given key features, assumptions must be made. As in this example, the same key features could describe many different graphs. The key features could also be represented by a parabola, a curve that is narrower or wider, or an absolute value function.

Think About It!

Explain why the end behavior is not defined in the context of this situation.

Sample answer: Because Hae cannot have a negative speed or time, the graph cannot extend into other quadrants.

Think About It!

Based on the graph, the speed of the car at 10 seconds is 40 miles per hour. Is it appropriate to assume that the car is traveling that exact speed at a specific time? Explain.

No; sample answer: because the graph only represents an approximation of the speed of the car over time, it is not possible to determine a specific speed at a specific time.

Example 2 Sketch a Nonlinear Function

Use the key features of the function to sketch its graph.

***y*-intercept:** (0, 3)

Linearity: nonlinear

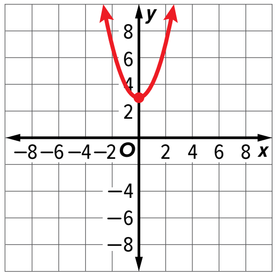
Continuity: continuous

Positive: for all values of x

Decreasing: for all values of x such that $x < 0$

Extrema: minimum at (0, 3)

End Behavior: As $x \to \infty$, $f(x) \to \infty$.
As $x \to -\infty$, $f(x) \to \infty$.



Example 3 Sketch a Real-World Function

TEST DRIVE **Hae is test driving a car she is thinking of buying. She decides to accelerate to 60 miles per hour and then decelerate to a stop to test its acceleration and brakes. It takes her 15 seconds to reach her maximum speed and 15 additional seconds to come to a stop. Use the key features to sketch a graph that shows the speed *y* as a function of time *x*.**

***y*-intercept:** Hae starts her test drive at a speed of 0 miles per hour.

Linear or Nonlinear: The function that models the situation is nonlinear.

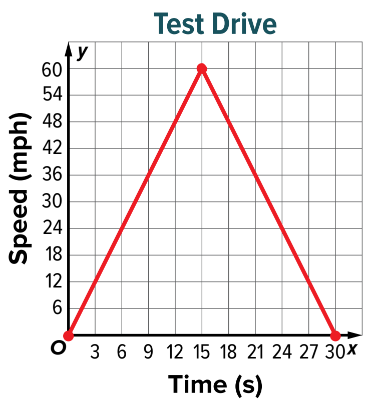
Extrema: Hae's maximum speed is 60 miles per hour, which she reaches 15 seconds into her test drive.

Increasing: Hae __increases__ the speed at a uniform rate for the first 15 seconds.

Decreasing: Hae decreases the speed at a __uniform__ rate for the next 15 seconds until she reaches a __stop__.

End Behavior: Because Hae starts at __0__ miles per hour and ends at __0__ miles per hour, there is __no__ end behavior.

Before sketching, consider the constraints of the situation. Hae cannot drive a negative speed or for a negative amount of time. Therefore, the graph only exists for positive x- and y-values.



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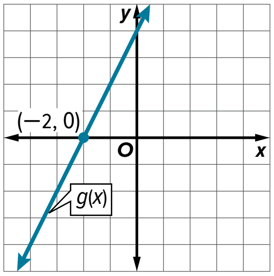
Example 4 Compare Properties of Linear Functions

Use the table and graph to compare the two functions.

x	$f(x)$
−6	−3
−3	−2
0	−1
3	0
6	1

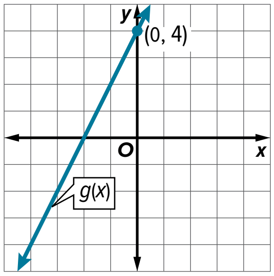


x-intercept of $f(x)$: 3

x-intercept of $g(x)$: −2.

So, $f(x)$ intersects the x-axis at a point farther to the right than $g(x)$.

x	$f(x)$
−6	−3
−3	−2
0	−1
3	0
6	1

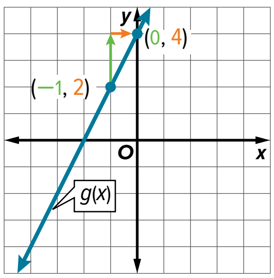


y-intercept of $f(x)$: −1,

y-intercept of $g(x)$: 4.

So, $g(x)$ intersects the y-axis at a higher point than $f(x)$.

x	$f(x)$
−6	−3
−3	−2
0	−1
3	0
6	1



slope of $f(x)$: $\frac{1}{3}$

slope of $g(x)$: 2

Each function is increasing, but the slope of $g(x)$ is greater than the slope of $f(x)$.

So, $g(x)$ increases faster than $f(x)$.

Think About It!

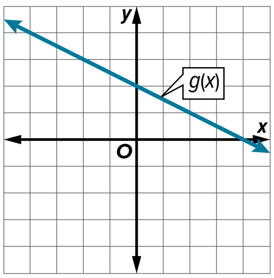
How would a function that passes through (1, 0) with a slope of −4 compare to $f(x)$ and $g(x)$?

Sample answer: The x-intercept would be between the x-intercepts of $f(x)$ and $g(x)$. The function is decreasing instead of increasing, but it has a faster rate of change than either $f(x)$ or $g(x)$.

Check

Use the table and graph to compare the two functions.

x	$f(x)$
−2	−6
−1	−4
0	−2
1	0
2	2



Which statements about $f(x)$ and $g(x)$ are true? Select all that apply.

- [ ] $g(x)$ has a faster rate of change than $f(x)$.
- [x] The x-intercept of $g(x)$ is greater than the x-intercept of $f(x)$.
- [ ] The y-intercept of $f(x)$ is greater than the x-intercept of $g(x)$.
- [ ] Both functions are decreasing.
- [x] $f(x)$ increases while $g(x)$ decreases.
- [x] $f(x)$ has a faster rate of change than $g(x)$.

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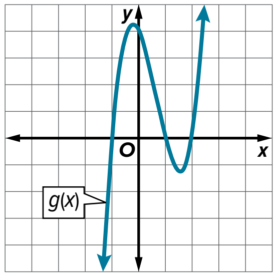
Example 5 Compare Properties of Nonlinear Functions

Examine the categories to see how to use the description and the graph to identify key features of each function. Then complete the statements to compare the two functions.

$f(x)$	$g(x)$
x-intercept: $(-3.4, 0)$ y-intercept: $(0, 1.5)$ relative maximum: $(-2.3, 4.7)$ relative minimum: $(-0.4, 1.1)$ end behavior: as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$	
x-intercept(s)	
$f(x)$ intersects the x-axis once at $(-3.4, 0)$.	$g(x)$ intersects the x-axis three times at $(-1, 0)$, $(1, 0)$, and $(2, 0)$.
y-intercept	
$f(x)$ intersects the y-axis at $(0, 1.5)$.	$g(x)$ intersects the y-axis at $(0, 4)$.
Extrema	
$f(x)$ has a relative maximum of 4.7 and a relative minimum of 1.1.	$g(x)$ has a relative maximum of about 4.2 and a relative minimum of about -1.2.
End Behavior	
As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$.	As $x \to -\infty$, $g(x) \to -\infty$, and as $x \to \infty$, $g(x) \to \infty$.

- The x-intercept of $f(x)$ is less than any of the x-intercepts of $g(x)$.
- The graph of $g(x)$ intersects the x-axis more times than $f(x)$.
- The y-intercept of $f(x)$ is less than the y-intercept of $g(x)$.
- So, $g(x)$ intersects the y-axis at a higher point than $f(x)$.
- The relative maximum of $f(x)$ is greater than the relative maximum of $g(x)$. The relative minimum of $f(x)$ is greater than the relative minimum of $g(x)$.
- The two functions have the same end behavior.

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