Linearity, Intercepts, and Symmetry

Explore Symmetry and Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you tell whether the graph of a function is symmetric?

Learn Linear and Nonlinear Functions

In a **linear function**, no variable is raised to a power other than 1. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers. Linear functions can be represented by **linear equations**, which can be written in the form $Ax + By = C$. The graph of a linear equation is a straight line.

A function that is not linear is called a **nonlinear function**. The graph of a nonlinear function includes a set of points that cannot all lie on the same line. A nonlinear function cannot be written in the form $f(x) = mx + b$. A **parabola** is the graph of a quadratic function, which is a type of nonlinear function.

Example 1 Identify Linear Functions from Equations

Determine whether each function is a linear function. Justify your answer.

a. $f(x) = \dfrac{6x - 5}{3}$

$f(x) = \dfrac{6x - 5}{3}$ Original equation

$f(x) = \dfrac{6}{3}x - \dfrac{5}{3}$ Distribute the denominator of 3.

$f(x) = 2x - \dfrac{5}{3}$ Simplify.

The function __can__ be written in the form $f(x) = mx + b$, so it __is__ a linear function.

b. $5y = 4 + 3x^3$

$5y = 4 + 3x^3$ Original equation

$5y = \underline{3x^3} + 4$ Commutative Property

The function __cannot__ be written in the form $f(x) = mx + b$ because the independent variable x is raised to a whole number power __greater than__ 1. So, it is a __nonlinear__ function.

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Today's Goals

- Identify linear and nonlinear functions.
- Identify and interpret the intercepts of functions.
- Identify whether graphs of functions possess line or point symmetry and determine whether functions are even, odd, or neither.

Today's Vocabulary

linear function
linear equation
nonlinear function
parabola
intercept
x-intercept
y-intercept
symmetry
line symmetry
line of symmetry
point symmetry
point of symmetry
even functions
odd functions

Study Tip

Linear Functions To write any linear equation in function form, solve the equation for y and replace the variable y with $f(x)$.

Your Notes

Think About It!

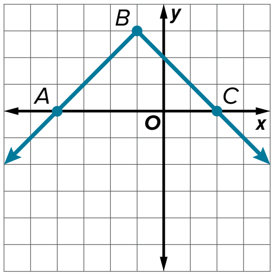
Why is $f(x) = \sqrt{2x} + 3$ not a linear function?

Sample answer: The variable is under the square root sign, which is the same as having an exponent of $\frac{1}{2}$.

Example 2 Identify Linear Functions from Graphs

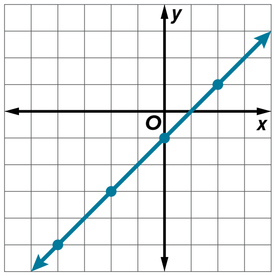
Determine whether each graph represents a *linear* or *nonlinear* function.

a.



There is no straight line that will contain the chosen points A, B, and C, so this graph represents a __nonlinear__ function.

b.



The points on this graph all lie on the same line, so this graph represents a __linear__ function.

Think About It!

Are negative x- or y-values possible in the context of the situation?

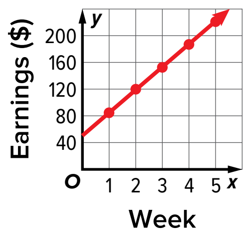
No; sample answer: because Makayla cannot work a negative number of weeks or earn a negative amount of money, the function only exists for nonnegative x- and y-values.

Example 3 Identify Linear Functions from Tables

EARNINGS **Makayla has started working part-time at the local hardware store. Her time at work steadily increases for the first five weeks. The table shows her total earnings each of those weeks. Are her weekly earnings modeled by a *linear* or *nonlinear* function?**

Week	1	2	3	4	5
Earnings ($)	85	119	153	187	221

Graph the points that represent the week and total earnings and try to draw a line that contains all the points.



Since there is a line that contains all the points, Makayla's earning can be modeled by a __linear__ function.

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Learn Intercepts of Graphs of Functions

A point at which the graph of a function intersects an axis is called an **intercept**. An ***x*-intercept** is the *x*-coordinate of a point where the graph crosses the *x*-axis, and a ***y*-intercept** is the *y*-coordinate of a point where the graph crosses the *y*-axis.

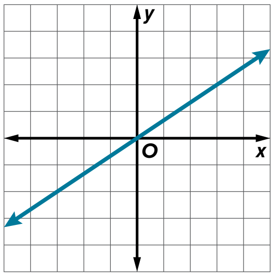
A linear function has at most one *x*-intercept while a nonlinear function may have more than one *x*-intercept.

Study Tip

Point or Coordinate *Intercept* may refer to the point or one of its coordinates. The context of the situation will often dictate which form to use.

Example 4 Find Intercepts of a Linear Function

Use the graph to estimate the *x*- and *y*-intercepts.



The graph intersects the *x*-axis at

(_0_, _0_), so the *x*-intercept is _0_.

The graph intersects the *y*-axis at

(_0_, _0_), so the *y*-intercept is _0_.

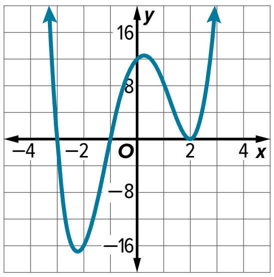
Think About It!

Describe a line that does not have two distinct intercepts.

Sample answer: A horizontal line has only a *y*-intercept. A vertical line has only an *x*-intercept. A line through the origin has the same *x*- and *y*-intercepts.

Example 5 Find Intercepts of a Nonlinear Function

Use the graph to estimate the *x*- and *y*-intercepts.



The graph appears to intersect the *x*-axis at (−3, 0), (−1, 0), and (2, 0), so the function has *x*-intercepts of _−3_, _−1_, and _2_.

The graph appears to intersect the *y*-axis at (_0_, _12_), so the function has a *y*-intercept of _12_.

Think About It!

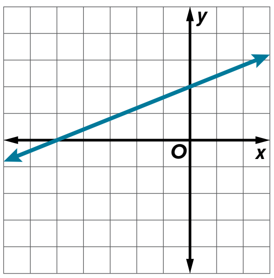
The graph of the nonlinear function has three *x*-intercepts. Can the graph have more than one *y*-intercept? Explain your reasoning.

Sample answer: No; if there is more than one *y*-intercept, then the graph would not pass the vertical line test. Thus, it would not be a function.

Check

Estimate the *x*- and *y*-intercepts of each graph.

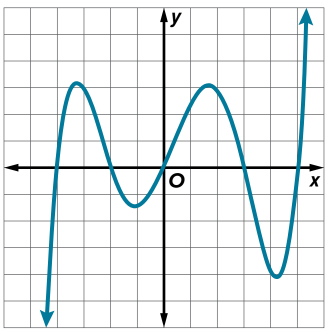
a.



x-intercept(s): _−5_

y-intercept(s): _2_

b.



x-intercept(s): _−4, −2, 0, 3, 5_

y-intercept(s): _0_

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Example 6 Interpret the Meaning of Intercepts

MODEL ROCKETS Ricardo launches a rocket from a balcony. The table shows the height of the rocket after each second of its flight.

Time (s)	Height (ft)
0	15
1	60
2	130
3	180
4	210
5	170
6	110
7	55
8	0

Part A Identify the *x*- and *y*-intercepts of the function that models the flight of the rocket.

In the table, the *x*-coordinate when $y = 0$ is 8. Thus, the *x*-intercept is 8.

In the table, the *y*-coordinate when $x = 0$ is 15. Thus, the *y*-intercept is 15.

Part B What is the meaning of the intercepts in the context of the rocket's flight?

The *x*-intercept is the number of seconds after the rocket is launched that it returns to the ground. The *y*-intercept is the height of the balcony from which the rocket is launched.

Think About It!

Describe the domain of the function that models the rocket's height over time.

Sample answer: The domain is $0 \leq x \leq 8$ and represents the amount of time that the rocket is in the air.

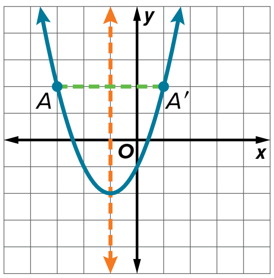
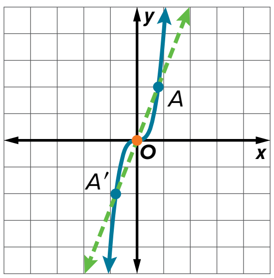
Watch Out!

Switching Coordinates A common mistake is to switch the coordinates for the intercepts. Remember that for the *x*-intercept, the *y*-coordinate is 0, and for the *y*-intercept, the *x*-coordinate is 0.

Learn Symmetry of Graphs of Functions

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself.

Key Concept • Symmetry

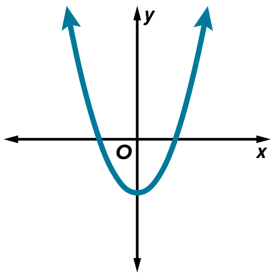
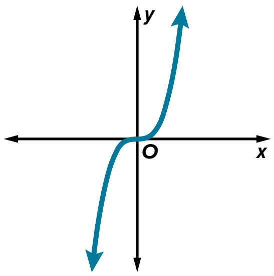
Type of Symmetry	Description	Example
A graph has **line symmetry** if it can be reflected in a line so that each half of the graph maps exactly to the other half.	The line dividing the graph into matching halves is called the **line of symmetry**. Each point on one side is reflected in the line to a point equidistant from the line on the opposite side.	
A graph has **point symmetry** when a figure is rotated 180° about a point and maps exactly onto the other part.	The point about which the graph is rotated is called the **point of symmetry**. The image of each point on one side of the point of symmetry can be found on a line through the point of symmetry equidistant from the point of symmetry.	

Talk About It

Can the graph of a function be symmetric in a horizontal line? Justify your answer.

Sample answer: Only if the graph of the function is a horizontal line; a horizontal line would be symmmetric with respect to itself. However if a graph other than a horizontal line were symmetric about a horizontal line, then the graph would contain two points with the same *x*-value. The graph would fail the vertical line test. Thus, it could not be a function.

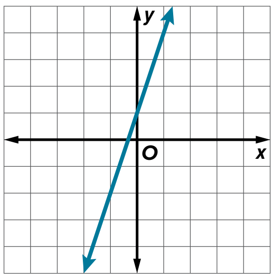
Key Concept • Even and Odd Functions

Type of Function	Algebraic Test	Example
Functions that have line symmetric with respect to the *y*-axis are called **even functions**.	For every x in the domain of f, $f(-x) = f(x)$.	
Functions that have point symmetry with respect to the origin are called **odd functions**.	For every x in the domain of f, $f(-x) = -f(x)$.	

Example 7 Identify Types of Symmetry

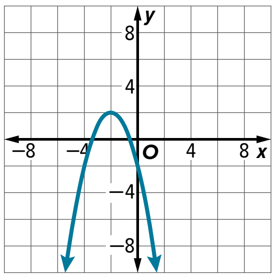
Identify the type of symmetry in the graph of each function. Explain.

a. $f(x) = 3x + 1$



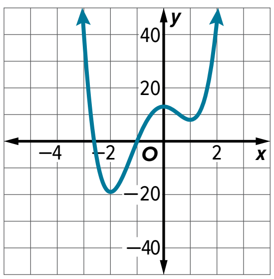
point symmetry: A 180° rotation about any point on the graph is the original graph.

b. $g(x) = -x^2 - 4x - 2$



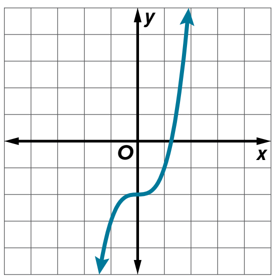
line symmetry: The reflection in the line $x = -2$ coincides with the original graph.

c. $h(x) = 3x^4 + 4x^3 - 12x^2 + 13$



no symmetry: There is no line or point of symmetry.

d. $j(x) = x^3 - 2$



point symmetry: A 180° rotation about the point (0, −2) is the original graph.

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Think About It!

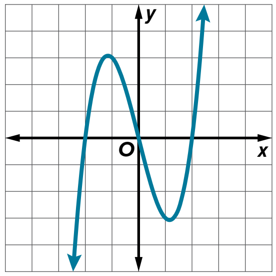
How would knowing the type of symmetry help you graph a function?

Sample answer: When you plot a point on one side of the line of symmetry or point of symmetry, you can plot its image by reflecting in the line or rotating about the point.

Example 8 Identify Even and Odd Functions

Determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If the function is odd or even, describe the symmetry.

a. $f(x) = x^3 - 4x$



It appears that the graph of $f(x)$ is symmetric about the origin. Substitute $-x$ for x to test this algebraically.

$f(-x) = (\underline{-x})^3 - 4(\underline{-x})$

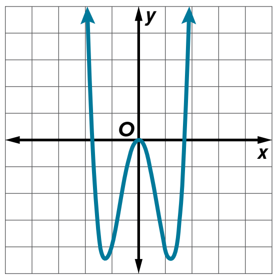
$= -x^3 + \underline{4x}$ Simplify.

$= -(x^3 - 4x)$ Distribute.

$= \underline{-f(x)}$ $f(x) = x^3 - 4x$

Because $f(-x) = -f(x)$ the function is <u>odd</u> and is symmetric about the <u>origin</u>.

b. $g(x) = 2x^4 - 6x^2$



It appears that the graph of $g(x)$ is symmetric about the y-axis. Substitute $-x$ for x to test this algebraically.

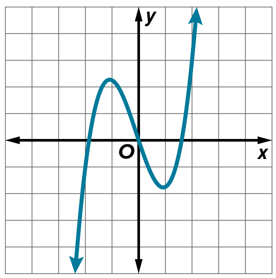
$g(-x) = 2(\underline{-x})^4 - 6(\underline{-x})^2$

$= 2x^4 - \underline{6x^2}$ Simplify.

$= g(x)$ $g(x) = 2x^4 - 6x^2$

Because $g(-x) = g(x)$ the function is <u>even</u> and is symmetric in the <u>y-axis</u>.

c. $h(x) = x^3 + 0.25x^2 - 3x$



It appears that the graph of $h(x)$ may be symmetric about the origin. Substitute $-x$ for x to test this algebraically.

$h(-x) = (\underline{-x})^3 + 0.25(\underline{-x})^2 - 3(\underline{-x})$

$= \underline{-x^3} + 0.25x^2 + \underline{3x}$

Because $-h(x) = -x^3 + 0.25x^2 + 3x$, the function is <u>neither even nor odd</u> because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$.

Watch Out!

Even and Odd Functions Always confirm symmetry algebraically. Graphs that appear to be symmetric may not actually be.

Check

Assume that f is a function that contains the point $(2, -5)$. Which of the given points must be included in the function if f is:

even? <u>$(-2, -5)$</u> odd? <u>$(-2, 5)$</u>

$(-2, -5)$ $(-2, 5)$ $(2, 5)$ $(-5, -2)$ $(-5, 2)$

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