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10-5 Study Guide and Intervention

Base e and Natural Logarithms

Base e and Natural Logarithms The irrational number $e \approx 2.71828\ldots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

Natural Base e	As n increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\ldots$. $\ln x = \log_e x$

The functions $y = e^x$ and $y = \ln x$ are inverse functions.

Inverse Property of Base e and Natural Logarithms	$e^{\ln x} = x$ $\quad \ln e^x = x$

Natural base expressions can be evaluated using the e^x and ln keys on your calculator.

Example 1 **Evaluate ln 1685.**

Use a calculator.

$\ln 1685 \approx 7.4295$

Example 2 **Write a logarithmic equation equivalent to $e^{2x} = 7$.**

$e^{2x} = 7 \rightarrow \log_e 7 = 2x$ or $2x = \ln 7$

Example 3 **Evaluate ln e^{18}.**

Use the Inverse Property of Base e and Natural Logarithms.

$\ln e^{18} = 18$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. ln 732 **2.** ln 84,350 **3.** ln 0.735 **4.** ln 100

5. ln 0.0824 **6.** ln 2.388 **7.** ln 128,245 **8.** ln 0.00614

Write an equivalent exponential or logarithmic equation.

9. $e^{15} = x$ **10.** $e^{3x} = 45$ **11.** $\ln 20 = x$ **12.** $\ln x = 8$

13. $e^{-5x} = 0.2$ **14.** $\ln (4x) = 9.6$ **15.** $e^{8.2} = 10x$ **16.** $\ln 0.0002 = x$

Evaluate each expression.

17. $\ln e^3$ **18.** $e^{\ln 42}$ **19.** $e^{\ln 0.5}$ **20.** $\ln e^{16.2}$

Lesson 10-5

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10-5 Study Guide and Intervention *(continued)*

Base e and Natural Logarithms

Equations and Inequalities with e and ln All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

Example **Solve each equation or inequality.**

a. $3e^{2x} + 2 = 10$

$3e^{2x} + 2 = 10$	Original equation
$3e^{2x} = 8$	Subtract 2 from each side.
$e^{2x} = \frac{8}{3}$	Divide each side by 3.
$\ln e^{2x} = \ln \frac{8}{3}$	Property of Equality for Logarithms
$2x = \ln \frac{8}{3}$	Inverse Property of Exponents and Logarithms
$x = \frac{1}{2} \ln \frac{8}{3}$	Multiply each side by $\frac{1}{2}$.
$x \approx 0.4904$	Use a calculator.

b. $\ln (4x - 1) < 2$

$\ln (4x - 1) < 2$	Original inequality
$e^{\ln (4x - 1)} < e^2$	Write each side using exponents and base *e*.
$0 < 4x - 1 < e^2$	Inverse Property of Exponents and Logarithms
$1 < 4x < e^2 + 1$	Addition Property of Inequalities
$\frac{1}{4} < x < \frac{1}{4}(e^2 + 1)$	Multiplication Property of Inequalities
$0.25 < x < 2.0973$	Use a calculator.

Exercises

Solve each equation or inequality.

1. $e^{4x} = 120$

2. $e^x \leq 25$

3. $e^{x-2} + 4 = 21$

4. $\ln 6x \geq 4$

5. $\ln (x + 3) - 5 = -2$

6. $e^{-8x} \leq 50$

7. $e^{4x-1} - 3 = 12$

8. $\ln (5x + 3) = 3.6$

9. $2e^{3x} + 5 = 2$

10. $6 + 3e^{x+1} = 21$

11. $\ln (2x - 5) = 8$

12. $\ln 5x + \ln 3x > 9$

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