


Operations on Functions

Explore Adding Functions

 **Online Activity** Use graphing technology to complete the Explore.

 **INQUIRY** Do you think that the graph of $f(x) + g(x)$ will be more or less steep than the graphs of $f(x)$ and $g(x)$?

Today's Goals

- Find sums, differences, products, and quotients of functions.
- Find compositions of functions.

Today's Vocabulary
composition of functions

Learn Operations on Functions

Key Concept • Operations on Functions

Operation	Definition	Example: Let $f(x) = 3x$ and $g(x) = 2x - 4$.
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 3x + (2x - 4)$ $= 5x - 4$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 3x - (2x - 4)$ $= x + 4$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$	$(f \cdot g)(x) = 3x(2x - 4)$ $= 6x^2 - 12x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$\left(\frac{f}{g}\right)(x) = \frac{3x}{2x - 4}, x \neq 2$

To graph the sum or difference of functions, graph each function separately. Then add or subtract the corresponding functional values.

Example 1 Add and Subtract Functions

Given $f(x) = -x^2 + 3x + 1$ and $g(x) = 2x^2 - 5$, find each function.

a. $(f + g)(x)$

$$(f + g)(x) = f(x) + \underline{\hspace{2cm}}$$

$$= (-x^2 + 3x + 1) + (2x^2 - 5)$$

$$= -x^2 + 3x + 1 + 2x^2 - 5$$

$$= x^2 + \underline{\hspace{1cm}}x - \underline{\hspace{1cm}}$$

Addition of functions

$$f(x) = -x^2 + 3x + 1$$

$$\text{and } g(x) = 2x^2 - 5$$

Add.

Simplify.

(continued on the next page)

 **Go Online** You can complete an Extra Example online.

Study Tip

Degree If the degree of $f(x)$ is m and the degree of $g(x)$ is n , then the degrees of $(f + g)(x)$ and $(f - g)(x)$ can be at most m or n , whichever is greater.

b. $(f - g)(x)$

$$(f - g)(x) = f(x) - \underline{\hspace{2cm}}$$

$$= (\underline{\hspace{2cm}}) - (2x^2 - 5)$$

$$= -x^2 + 3x + 1 - 2x^2 + 5$$

$$= -3x^2 + \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$$

Subtraction of functions

$$f(x) = -x^2 + 3x + 1 \text{ and } g(x) = 2x^2 - 5$$

Subtract.

Simplify.

Example 2 Multiply and Divide Functions

Given $f(x) = 4x^2 - 2x + 3$ and $g(x) = -x + 5$, find each function.

a. $(f \cdot g)(x)$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Multiplication of functions

$$= (4x^2 - 2x + 3)(\underline{\hspace{2cm}})$$

$$f(x) = 4x^2 - 2x + 3 \text{ and } g(x) = -x + 5$$

$$= -4x^3 + \underline{\hspace{1cm}}x^2 + \underline{\hspace{1cm}}x^2 - 10x - \underline{\hspace{1cm}}x + 15 \quad \text{Distributive Property}$$

$$= -4x^3 + \underline{\hspace{1cm}}x^2 - \underline{\hspace{1cm}}x + 15$$

Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Division of functions

$$= \frac{4x^2 - 2x + 3}{-x + 5}, x \neq \underline{\hspace{1cm}}$$

$$f(x) = 4x^2 - 2x + 3 \text{ and } g(x) = -x + 5$$

Check

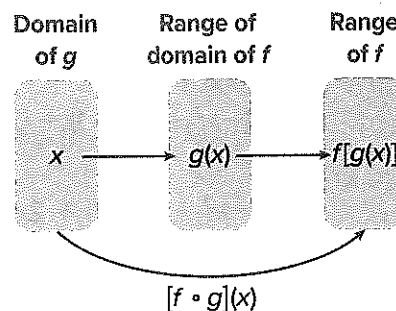
Given $f(x) = -x^2 + 1$ and $g(x) = x + 1$, find each function.

$$(f \cdot g)(x) = \underline{\hspace{2cm}} \quad \left(\frac{f}{g}\right)(x) = \underline{\hspace{2cm}}$$

Learn Compositions of Functions

Key Concept • Composition of Functions

Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composition function $f \circ g$ can be described by $[f \circ g](x) = f[g(x)]$.



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Example 4 Compose Functions by Using Ordered Pairs

Given f and g , find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

$$f = \{(1, 12), (10, 11), (0, 13), (9, 7)\}$$

$$g = \{(4, 1), (5, 0), (13, 9), (12, 10)\}$$

Part A Find $[f \circ g](x)$ and $[g \circ f](x)$.

To find $f \circ g$, evaluate $g(x)$ first then use the range to evaluate $f(x)$.

$$f[g(4)] = f(1) \text{ or } \underline{\quad\quad} \quad g(4) = 1$$

$$f[g(5)] = f(0) \text{ or } \underline{\quad\quad} \quad g(5) = 0$$

$$f[g(13)] = f(9) \text{ or } \underline{\quad\quad} \quad g(13) = 9$$

$$f[g(12)] = f(10) \text{ or } \underline{\quad\quad} \quad g(12) = 10$$

To find $g \circ f$, evaluate $f(x)$ first then use the range to evaluate $g(x)$.

$$g[f(1)] = g(12) \text{ or } \underline{\quad\quad} \quad f(1) = 12$$

$$g[f(10)] = g(11) \quad f(10) = 11$$

$$g[f(0)] = g(13) \text{ or } \underline{\quad\quad} \quad f(0) = 13$$

$$g[f(9)] = g(7) \quad f(9) = 7$$

Because 11 and 7 are not in the domain of g , $g \circ f$ is undefined for $x = 11$ and $x = 7$. So, $g \circ f = \{(1, 10), (0, 9)\}$.

Part B State the domain and range.

$[f \circ g](x)$: The domain is the x -coordinates of the composed function, so $D = \{\underline{\quad\quad}, 5, \underline{\quad\quad}, 13\}$. The range is the y -coordinates of the composed function, so $R = \{7, 11, \underline{\quad\quad}, \underline{\quad\quad}\}$.

$[g \circ f](x)$: The domain is the x -coordinates of the composed function, so $D = \{\underline{\quad\quad}\}$. The range is the y -coordinates of the composed function, so $R = \{9, 10\}$.

Example 5 Compose Functions

Given $f(x) = 2x - 5$ and $g(x) = 3x$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

Part A Find $[f \circ g](x)$ and $[g \circ f](x)$.

$$[f \circ g](x) = f[g(x)]$$

Composition
of functions

$$= \underline{\quad\quad}$$

Substitute.

$$= 2(3x) - 5 \quad \text{Substitute again.}$$

$$= 6x - 5 \quad \text{Simplify.}$$

$$[g \circ f](x) = g[f(x)]$$

$$= g(2x - 5)$$

$$= \underline{\quad\quad}$$

$$= 6x - 15$$

Part B State the domain and range.

Because $[f \circ g](x)$ and $[g \circ f](x)$ are both linear functions with nonzero slopes, $D = \{\underline{\quad\quad}\}$ and $R = \{\text{all real numbers}\}$ for both functions.

 **Go Online** You can complete an Extra Example online.

Study Tip

Domain and Range To ensure you have the right domain and range, it can help to graph $[f \circ g](x)$ and $[g \circ f](x)$.

Check

Given $f(x) = -x + 1$ and $g(x) = 2x^3 - x$, find $[f \circ g](x)$ and $[g \circ f](x)$. State the domain and range for each.

$$[f \circ g](x) = \underline{\hspace{2cm}} \qquad [g \circ f](x) = \underline{\hspace{2cm}}$$

Domain of $[f \circ g](x)$: Domain of $[g \circ f](x)$:

Range of $[f \circ g](x)$: Range of $[g \circ f](x)$:

Apply Example 6 Use Composition of Functions

BOX OFFICE A movie theater charges \$8.50 for each of the x tickets sold. The manager wants to determine how much the movie theater gets to keep of the ticket sales if they have to give the studios 75% of the money earned on ticket sales $t(x)$. If the amount they keep of each ticket sale is $k(x)$, which composition represents the total amount of money the theater gets to keep?

1 What is the task?

Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

2 How will you approach the task? What have you learned that you can use to help you complete the task?

3 What is your solution?


Use your strategy to solve the problem.

What function represents the money earned on ticket sales, $t(x)$?

What function represents the amount of money the theater keeps from each ticket sale, $k(x)$?

Because the theater uses the total earnings to determine the amount they keep from the ticket sales, what composition should be used to represent the situation?

4 How can you know that your solution is reasonable?

 **Write About It!** Write an argument that can be used to defend your solution.

Watch Out!

Order Remember that, for two functions $f(x)$ and $g(x)$, $[f \circ g](x)$ is not always equal to $[g \circ f](x)$. Given that the studios take their cut after the tickets have been sold, consider how that affects the order of $t(x)$ and $k(x)$.



Go Online

You can complete an Extra Example online.