**5.5 Roots and Zeros ⸱ Form A**

**All work must be completed in a clear and organized manner on a separate sheet of paper. Final answers only, boxed in on this sheet.**

**Example 1**

**Solve each equation. State the number and type of roots.**

 **1.** $x^{2}$ – 4*x* + 40 = 0 **2.** $x^{4}$ – 625 = 0

 **3.** $x^{5}$ – 81*x* = 0 **4.** 4*x*2 + 1 = 0

 **5.** 2*x*2 – 5*x* + 14 = 0 **6.** 8*x*3 – 27 = 0

 **7.** *x*3 – 6*x*2 + 7*x* = 0 **8.** *x*5 + 2*x*3 + *x* = 0

**Example 2**

**State the possible number of positive real zeros, negative real zeros, and imaginary zeros of
each function.**

 **9.** *h*(*x*) = 4$x^{3}$ – 12$x^{2}$ – *x* + 3 **10.** *p*(*x*) = $x^{3}$ – $x^{2}$ + 4*x* – 6

 **11.** *f*(*x*) = $x^{4}$ – $x^{3}$ – 5$x^{2}$ + 6*x* + 1 **12.** *f*(*x*) = 2*x*3 – 7*x*2 – 2*x* + 12

 **13.** *f*(*x*) = *x*4 – 2*x*2 – 5*x* + 19 **14.** *f*(*x*) = –*x*5 + 14*x*3 + 18*x* – 36

**Example 3**

**Find all of the zeros of each function and use them to sketch a rough graph.**

 **15.** *g*(*x*) = $x^{3}$ – 6$x^{2}$ + 13*x* – 10 **16.** *q*(*x*) = $x^{3}$ + 3$x^{2}$ – 6*x* – 8

 **17.** *f*(*x*) = $x^{4}$ – 21$x^{2}$ + 80 **18.** *f*(*x*) = *x*3 + *x*2 – 17*x* + 15

 **19.** *f*(*x*) = *x*4 + 6*x*3 + 73*x*2 + 384*x* + 576 **20.** *f*(*x*) = *x*5 – 8*x*3 – 9*x*

**Example 4**

**Write a polynomial that could be represented by each graph.**

 **21.**

**Example 5**

 **22. FISH** Some fish jump out of the water. When a fish is out of the water, it’s location is above sea level. When a fish dives back into the water, it’s location is below sea level. A biologist can use polynomial functions to model the location of fish compared to see level. A biologist noticed that a fish is at sea level at –3, –2, –1, 1, 2, and 3 minutes from noon. Graph a polynomial function that could represent the location of the fish compared to sea level *y*, in centimeters, *x* seconds from noon.

**Mixed Exercises**

**Write a polynomial function of least degree with integral coefficients that has the given zeros.**

 **23.** 5, –2, –1 **24.** –1, –1, 2*i*

 **25.** 0, –5, 3 + *i*

**Sketch the graph of each function using its zeros.**

 **26.** *f*(*x*) = *x*3 – 5*x*2 – 2*x* + 24 **27.** *f*(*x*) = *x*4 – 6*x*3 + 7*x*2 + 6*x* – 8

 

**28. USE A SOURCE** Linear algebra is the study of linear equations. In linear algebra, the coefficients of linear equations are often organized into rectangular arrays called matrices. Research the eigenvalues of a matrix and how they relate to the roots of a polynomial function. What fields use linear algebra, matrices, and eigenvalues?.

**29. ANALYZE** Use the zeros to draw the graph of *P*(*x*) = *x*3 – 7*x*2 + 7*x* + 15 by hand. Discuss the accuracy of your graph, and what could be done to improve the accuracy.

**30. CREATE** Sketch the graph of a polynomial function with:

 **a.** 3 real, 2 imaginary zeros **b.** 4 real zeros **c.** 2 imaginary zeros

**31. WHICH ONE DOESN’T BELONG?** Determine which equation is not like the others. Justify your conclusion.



**32. WRITE** Explain to a friend how you would use Descartes’ Rule of Signs to determine the number of possible positive real roots and the number of possible negative real roots of the polynomial function *f*(*x*) = *x*4 – 2*x*3 + 6*x*2 + 5*x* – 12.