**5.4 The Remainder and Factor Theorems ⸱ Form A**

**All work must be completed in a clear and organized manner on a separate sheet of paper. Final answers only, boxed in on this sheet.**

**Example 1**

**Use synthetic substitution to find *f*(–5) and *f*(2) for each function.**

 **1.** *f*(*x*) = *x*2 – 8*x* + 6 **2.** *f*(*x*) = 2*x*3 – 8*x*2 – 2*x* + 5

 **3.** *f*(*x*) = *x*5 + 8*x*3 + 2*x* – 15 **4.** *f*(*x*) = *x*4 – 6*x* – 8

**Use synthetic substitution to find *f*(2) and *f*(–1) for each function.**

 **5.** *f*(*x*) = *x*2 – *x* + 1 **6.** *f*(*x*) = *x*3 + 2*x*2 + 5

 **7.** *f*(*x*) = *x*3 + 6*x*2 + *x* – 4 **7.** *f*(*x*) = *x*3 – 5*x*2 – *x* + 6

 **8.** *f*(*x*) = *x*4 – 3*x*3 + 2*x*2 – 2*x* + 6 **9.** *f*(*x*) = *x*6 – 2*x*5 + *x*4 + *x*3 – 9*x*2 – 20

**Example 2**

**10. PROFIT** The profit, in thousands, of Clyde’s Corporation can be modeled by
*P*(*y*) = $y^{4} $– 4$y^{3} $+ 2$y^{2} $+ 10*y* – 200, where *y* is the number of years after the business was started. Predict the profit of Clyde’s Corporation after 10 years.

**Example 3**

**Given a polynomial and one of its factors, find the remaining factors of the polynomial.**

 **11.** *x*4 + 2*x*3 – 8*x* – 16; *x* + 2 **12.** *x*3 – *x*2 – 5*x* – 3; *x* – 3

 **13.** 2*x*3 + 7*x*2 – 53*x* – 28; *x* – 4 **14. 3***x*3 – 19*x*2 – 15*x* + 7; *x* – 7

 **15.** *x*3 + *x*2 – 5*x* + 3; *x* – 1 **16.** *x*3 – 6*x*2 + 11*x* – 6; *x* – 3

 **17.** *x*3 – 6*x*2 + 32; *x* – 4 **18.** *x*3 – 19*x* + 30; *x* – 2

 **19.** 2*x*3 + *x*2 – 5*x* + 2; *x* + 2 **19.** 3*x*3+ *x*2 + *x* – 2; 3*x* – 2

 **21.** 16*x*5 – 32*x*4 – 81*x* + 162; 2*x* – 3

**Mixed Exercises**

**22. STATE YOUR ASSUMPTION** The revenue from streaming music services in the United States from 2005 to 2016 can be modeled by y = 0.26x5 – 7.48x4 + 79.20x3 – 333.33x2 + 481.68x + 99.13, where x is the number of years since 2005 and y is the revenue in millions of U.S. dollars.

**a.** Estimate the revenue from streaming music services in 2010.

**b.** What might the revenue from streaming music services be in 2020? What assumption did you make to make your prediction?

**Find values of *k* so that each remainder is 3.**

 **23.** (*x*2 – *x* + *k*) ÷ (*x* – 1) **24.** (*x*2 + 5*x* + 7) ÷ (*x* + *k*)

**25.** If *f*(–8) = 0 and *f*(*x*) = *x*3 – *x*2 – 58*x* + 112, find all the zeros of *f*(*x*) and use them to graph the function.

**26. GEOMETRY** The volume of a box with a square base is *V*(*x*) = 2*x*3 + 15*x*2 + 36*x* + 27. If the
height of the box is (2*x* + 3), what are the sides of the base in terms of *x*?

**27. CONSTRUCT ARGUMENTS** Divide the polynomial function *f*(*x*) = 4*x*3 – 10*x* + 8 by the factor

(*x* + 5). Then state and confirm the Remainder Theorem for this particular polynomial function
and factor.

**28. STRUCTURE** Verify the Remainder Theorem for the polynomial *x*2 + 3*x* + 5 and the factor
(*x* – ) by first using synthetic division and then evaluating for x = .

**29. REGULARITY** Polynomial *f*(*x*) is divided by *x* – *c*. What can you conclude if:

 **a.** the remainder is 0?

 **b.** the remainder is 1?

 **c.** the quotient is 1, and the remainder is 0?

 **30. CREATE** Write a polynomial function that has a double zero of 1 and a double zero of –5. Graph the function.

**31. ANALYZE** Review the definition for the Factor Theorem. Provide a proof of the theorem.

**32. PERSEVERE** Show that the quartic function *f*(*x*) = *ax*4 + *bx*3 + *cx*2 + *dx* + *e* will always have a rational zero when the numbers 1, –2, 3, 4, and –6 are randomly assigned to replace *a* through *e*, and all of the numbers are used.

**33. FIND THE ERROR** The table shows the *x*-values and their corresponding *P*(*x*) values for a polynomial function. Tyrone and Nia used the Factor Theorem to find all of the factors of *P*(*x*). Is either of them correct? Explain your reasoning.

