The Remainder and Factor Theorems

Explore Remainders

- Online Activity Use the interactive tool to complete the Explore.
 - INQUIRY How are the divisor and quotient of a polynomial related to its factors when the remainder is zero?

Learn The Remainder Theorem

Polynomial division can be used to find the value of a function. From the Division Algorithm, we know that $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$ and that $f(x) = q(x) \cdot g(x) + r(x)$, where q and r are unique and the degree of r is less than the degree of g. Suppose we were to call the dividend p(x)and the divisor x - a. Then the Division Algorithm would be $\frac{p(x)}{x-a} = q(x) + \frac{r}{x-a}$ and $p(x) = q(x) \cdot (x-a) + r$, where a is a constant and r is the remainder. Since any polynomial can be written in this form, evaluating p(x) at a gives the following.

$$p(x) = q(x) \cdot (x - a) + r$$

Polynomial function p(x)

$$p(a) = q(a) \cdot (a - a) + r$$

Substitute a for x.

$$p(a) = q(a) \cdot (0) + r$$

$$a - a = 0$$

$$p(a) = r$$

$$q(a) \cdot (0) = 0$$

This shows how the Remainder Theorem can be used to evaluate a polynomial at p(a).

Key Concept · Remainder Theorem

Words: For a polynomial p(x) and a number a, the remainder upon division by x - a is p(a).

Example: Evaluate $p(x) = x^2 - 4x + 7$ when x = 5.

Synthetic division

Direct substitution

$$p(x) = x^2 - 4x + 7$$

$$p(5) = 5^2 - 4(5) + 7$$

$$p(5) = 12$$

Today's Goals

- Evaluate functions by using synthetic substitution.
- · Use the Factor Theorem to determine factors of polynomials.

Today's Vocabulary synthetic substitution depressed polynomial

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| Your Notes | Applying the Remainder Theorem to evaluate a function is called synthetic substitution . You may find that synthetic substitution is a more convenient way to evaluate a polynomial function, especially when the degree of the function is greater than 2. | | | |
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| ter Control of the State (State Control of the State Control of the Stat | Example 1 Synthetic Substitution | | | |
| Study Tip Missing terms | Use synthetic substitution to find $f(-3)$ if $f(x) = -2x^4 + 3x^2 - 15x + 9$. | | | |
| Remember to include | By the Remainder Theorem, $f(-3)$ is the remainder of $\frac{f(x)}{x-(-3)}$. | | | |
| zeros as placeholders | _3 _2 0 3 _15 9 | | | |
| for any missing terms in the polynomial. | 6 -18 45 -90 | | | |
| | -2 | | | |
| Experimental interval of the property of the september of | The remainder is -81 . Therefore, $f(-3) = $ | | | |
| Hellali de vive-dem egenya, sangan pengada da damaken sanga Ninasanan sangan sangan sangan sangan sangan sanga | The remainder is —81. Therefore, i(3) — | | | |
| E. of PERSONNEL SERVICE SERVICES SERVIC | Use direct substitution to check. | | | |
| \$200-POLITAPANENE PARALES AND MENTAL PER FORM PARAMETER AND | $f(x) = -2x^4 + 3x^2 - 15x + 9$ Original function | | | |
| promovements of the contract o | $f(-3) = -2(-3)^4 + 3(-3)^2 - 15(-3) + 9$ Substitute -3 for x . | | | |
| | = -162 + 27 + 45 + 9 or -81 True | | | |
| | Check Use synthetic substitution to evaluate $f(x) = -6x^3 + 52x^2 - 27x - 31$. $f(8) = \underline{\qquad}$ | | | |
| Think About It! How could you use the function and synthetic | Example 2 Apply the Remainder Theorem EGG PRODUCTION The total production of eggs in billions in the United States can be modeled by the function | | | |
| substitution to estimate the number of eggs produced in 1990? | $f(x) = 0.007x^3 - 0.149x^2 + 1.534x + 84.755$, where x is the number of years since 2000. Predict the total production of eggs in 2025. | | | |
| What assumption would | Since $2025 - 2000 = 25$, use synthetic substitution to determine $f(25)$. | | | |
| you have to make to solve this problem? | 25 0.007 -0.149 1.534 84.755 | | | |
| LIABAGE TO THE PROPERTY OF THE | 0.175 0.65 54.6 | | | |
| | 0.007 | | | |
| | In 2025, approximately billion eggs will be produced in the United States. | | | |
| | Go Online You can complete an Extra Example online. | | | |

Check

KITTENS The ideal weight of a kitten in pounds is modeled by the function $f(x) = 0.009x^2 + 0.127x + 0.377$, where x is the age of the kitten in weeks. Determine the ideal weight of a 9-week-old kitten. Round to the nearest tenth.

____ pounds

Learn The Factor Theorem

When a binomial evenly divides a polynomial, the binomial is a factor of the polynomial. The quotient of this division is called a depressed polynomial. The **depressed polynomial** has a degree that is one less than the original polynomial.

A special case of the Remainder Theorem is called the Factor Theorem.

Key Concept • Factor Theorem

Words: The binomial x - a is a factor of the polynomial p(x) if and only if p(a) = 0.

Example:

dividend quotient divisor remainder
$$x^3 - x^2 - 30x + 72 = (x^2 - 7x + 12) \cdot (x + 6) + 0$$

 $x + 6$ is a factor of $x^3 - x^2 - 30x + 72$.

Example 3 Use the Factor Theorem

Show that x + 8 is a factor of $2x^3 + 15x^2 - 11x - 24$. Then find the remaining factors of the polynomial.

Because the remainder is 0, x + 8 _____ a factor of the polynomial by the Factor Theorem. So $2x^3 + 15x^2 - 11x - 24$ can be factored as $(x + 8)(2x^2 - x - 3)$. The depressed polynomial is ____. Check to see if this polynomial can be factored.

$$2x^2 - x - 3 = (_____)(x + 1)$$
 Factor the trinomial.
Therefore, $2x^3 + 15x^2 - 11x - 24 = (x + 8)(2x - 3)(_____)$.

Go Online

You may want to complete the Concept Check to check your understanding.

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Factoring Some depressed polynomials may not be factorable. In that case, the only factors are the divisor and the depressed polynomial.

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Select all of the factors of $3x^3 + 10x^2 - 27x - 10$.

A. x - 2

B. x + 5

C. x + 9

D. x - 10

E. 3x + 1

F. 3x - 10

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?



Go Online You can complete an Extra Example online.