Proving Polynomial Identities

Explore Polynomial Identities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you prove that two polynomial expressions form a polynomial identity?

Today's Goal
- Prove polynomial identities and use them to describe numerical relationships.

Today's Vocabulary
identity
polynomial identity

Learn Proving Polynomial Identities

An **identity** is an equation that is satisfied by any numbers that replace the variables. Thus, a **polynomial identity** is a polynomial equation that is true for any values that are substituted for the variables.

Unlike solving an equation, do not begin by assuming that an identity is true. You cannot perform the same operation to both sides and assume that equality is maintained.

Key Concept • Verifying Identities by Transforming One Side
- Simplify one side of an equation until the two sides of the equation are the same. It is often easier to transform the more complicated expression into the form of the simpler side.
- Factor or multiply expressions as necessary. Simplify by combining like terms.

Study Tip

Transforming One Side It is often easier to work with the more complicated side of an equation. Look at each side and determine which requires more steps to be simplified. For example, it is often easier to work on the side that involves the square or cube of an algebraic expression.

Example 1 Transform One Side

Prove that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

$x^3 - y^3 \stackrel{?}{=} (x - y)(x^2 + xy + y^2)$	Original equation
$\stackrel{?}{=} x(x^2) + x(xy) + x(y^2) - y(x^2) - y(xy) - y(y^2)$	Distributive Property
$\stackrel{?}{=}$ ___ + ___ $+ xy^2 -$ ___ $-$ ___ $- y^3$	Simplify.
$\stackrel{?}{=} x^3 + x^2y - x^2y + xy^2 - xy^2 - y^3$	Commutative Property
$=$ ___ $-$ ___	True

Because the expression on the right can be simplified to be the same as the expression on the left, this proves the polynomial identity.

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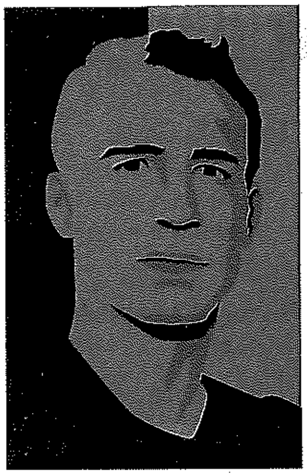
Talk About It!

If you multiplied each side of the equation by a variable z, would the result still be a polynomial identity? Explain your reasoning.

Your Notes



Math History Minute:

Former quarterback **Frank Ryan (1936–)** earned his Ph.D. in mathematics about six months after he led the Cleveland Browns to the NFL championship game of 1964, where they won 27-0. During part of his academic career, Ryan studied prime numbers, including Opperman's Conjecture that there is a prime number between n^2 and $n^2 + n$, where n is an integer. This work could eventually lead to a polynomial identity that could be used to identify prime numbers.

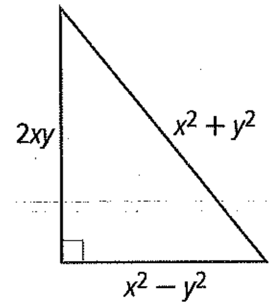
Use a Source

Research an application of prime numbers. How could a polynomial identity for identifying prime numbers impact the application?

Example 2 Use Polynomial Identities

TRIANGLES **Pedro claims that you can always create three lengths that form a right triangle by using the following method: take two positive integers x and y where $x > y$. Two legs of a right triangle are defined as $x^2 - y^2$ and $2xy$. The hypotenuse is defined as $x^2 + y^2$. Is Pedro correct? Explain your reasoning in the context of polynomial identities.**

To determine whether Pedro is correct, we can use information about right triangles and the expressions involving x and y to try to construct a polynomial identity. If $x^2 - y^2$ and $2xy$ are the legs of the triangle, and $x^2 + y^2$ is the hypotenuse, then it should be true that $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$.



If this is an identity, you can simplify the expressions for the sides to be the same expression.

$(x^2 - y^2)^2 + (2xy)^2 \stackrel{?}{=} (x^2 + y^2)^2$	Original equation
$x^4 - 2x^2y^2 + y^4 + 4x^2y^2 \stackrel{?}{=} x^4 + 2x^2y^2 + y^4$	Square each term.
$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$	True

Because the identity is ______, this proves that Pedro is correct. His process for creating the sides of a right triangle will always work.

Check

Write in the missing explanations to prove that $x^4 - y^4 \stackrel{?}{=} (x - y)(x + y)(x^2 + y^2)$.

$x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$	Original equation
$x^4 - y^4 \stackrel{?}{=} (x^2 - y^2)(x^2 + y^2)$	______
$x^4 - y^4 \stackrel{?}{=} x^4 + x^2y^2 - x^2y^2 - y^4$	______
$x^4 - y^4 = x^4 - y^4$	______

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