5.3 Proving Polynomial Identities · Form A

All work must be completed in a clear and organized manner on a separate sheet of paper. Final answers only, boxed in on this sheet.
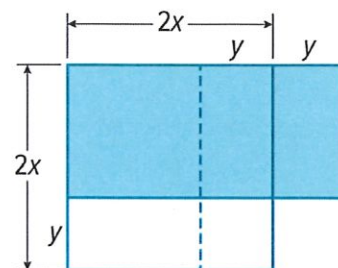
Example 1

Prove each polynomial identity. All work must be on a separate sheet, including the final solution.

1. $(x + 5)^2 = x^2 + 10x + 25$ _need the proof that these are equal._

2. $(2x^2 + y^2)^2 = (2x^2 - y^2)^2 + (2xy\sqrt{2})^2$

3. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

4. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

Example 2

5. **USE A MODEL** Julio claims that you can find the area of a rectangle using the following method: take two positive integers x and y, where $x > y$. The side lengths of the rectangle are defined by the expression $2x + y$ and $2x - y$. The area of the rectangle is defined by the expression $4x^2 - y^2$. Is Julio correct? Explain your reasoning in the context of polynomial identities. _yes, $(2x+y)(2x-y) = 4x^2 - y^2$_

Mixed Exercises

Determine whether each equation is an identity. All work must be on a separate sheet, including the final solution.

6. $(x + 2)(x + 1)^2 = (x^2 + 3x + 2)(x + 1)$

7. $(x + 2)^2(x^3 - 3x^2 + 3x - 1) = (x^2 + 4x + 4)(x - 1)^3$

need the proof that these are equal.

8. **USE TOOLS** Consider the following equation.

$$(x - 2)^2(x^3 + 9x^2 + 27x + 27) = (x^2 - 4x + 4)(x + 3)^3$$

a. Evaluate the expressions for each value to complete the table.

x	$(x - 2)^2(x^3 + 9x^2 + 27x + 27)$	$(x^2 - 4x + 4)(x + 3)^3$
0	108	108
1	64	64
2	0	0
3	216	216
4	1372	1372

b. What conclusion can you make about the equation, based on the results in your table? Explain. _They may be polynomial identities_

c. How can you prove your conclusion from **part b**? _use the sum of cubes rule._

~~USE TOOLS Use a computer algebra system (CAS) to prove each identity.~~

~~18. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$~~

~~20. $(x + 1)^2(x - 4)^3 = (x^2 - 3x - 4)(x^3 - 7x^2 + 8x + 16)$~~

21. **WRITE** Explain the meaning of polynomial identity and summarize the method for proving an equation is a polynomial identity. _Begin w/ the more complicated side_

23. **ANALYZE** Refer to Example 2. Notice that Pedro says x and y must be positive integers and x must be greater than y. Explain why these restrictions are necessary. _it is referring to measurement._

25. **FIND THE ERROR** George is proving the identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ by simplifying the right side. His work is shown. Is George's correct? If not, identify and correct his error. _Should not be negative._

$(a + b)(a^2 - ab + b^2)$
$= a^3 - a^2b + ab^2 \;(-a^2b - ab^2 + b^3$
$= a^3 - 2a^2b + b^3$