Solving Polynomial Equations by Graphing

Today's Goals
- Solve polynomial equations by graphing.

Explore Solutions of Polynomial Equations

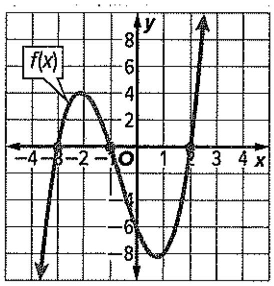
Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you solve a polynomial equation by using the graph of a related polynomial function?

Think About It!
How can you use the structure of the related function to determine the number of real solutions of the equation?

Learn Solving Polynomial Equations by Graphing

A related function is found by rewriting the equation with 0 on one side, and then replacing 0 with $f(x)$. The values of x for which $f(x) = 0$ are the real zeros of the function and the x-intercepts of its graph.



$x^3 + 2x^2 - 4x = x + 6$
- -3, -1, and 2 are solutions.
- -3, -1, and 2 are roots.

$f(x) = x^3 + 2x^2 - 5x - 6$
- -3, -1, and 2 are zeros.
- -3, -1, and 2 are x-intercepts.

Example 1 Solve a Polynomial Equation by Graphing

Use a graphing calculator to solve $x^4 + 3x^2 - 5 = -4x^3$ by graphing.

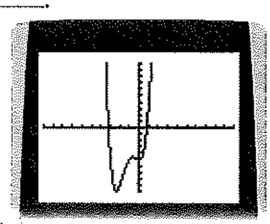
Step 1 Find a related function. Write the equation with 0 on the right.

$x^4 + 3x^2 - 5 = -4x^3$	Original equation
$x^4 + 3x^2 - 5 +$ ____ $= -4x^3 +$ ____	Add $4x^3$ to each side.
$x^4 + 4x^3 + 3x^2 - 5 =$ ____	Simplify.

A related function is $f(x) =$ ____________.

Step 2 Graph the related function.

Enter the equation in the **Y** = list and graph the function.



Step 3 Find the zeros.

Use the **zero** feature from the **CALC** menu. The real zeros are about ______ and ______.

Talk About It!
Explain how you could use the table feature to more accurately estimate the zeros of the related function. What are the limitations of the table feature?

Check

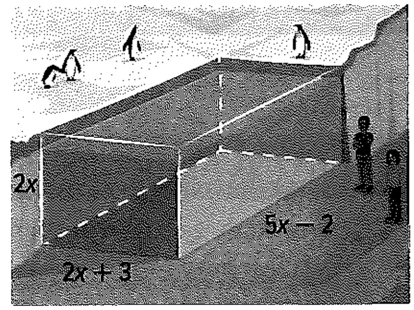
Use a graphing calculator to solve $4x^2 + x = \frac{1}{2}x^4 + 1$ by graphing. Round to the nearest hundredth, if necessary.

$x =$ ______________

Your Notes

Example 2 Solve a Polynomial Equation by Using a System

ANIMALS For an exhibit with six or fewer Emperor penguins, the pool must have a depth of at least 4 feet and a volume of at least 1620 gallons, or about 217 ft^3, per bird. If a zoo has five Emperor penguins, what should the dimensions of the pool shown at the right be to meet the minimum requirements?



Part A Write a polynomial equation.

Use the formula for the volume of a rectangular prism, $V = \ell wh$, to write a polynomial equation that represents the volume of the pool. Let h represent the depth of the pool.

Since the minimum required volume for the pool is ____ ft^3 per penguin, or ____ $\cdot 5 =$ ____ ft^3, the equation that represents the volume of the pool is $(2x + 3)(5x - 2)2x =$ ____. Simplify the equation.

$(2x + 3)(5x - 2)2x = 1085$		Volume of pool
$[2x(5x) + 2x(-2) + 3(5x) + 3(-2)]2x = 1085$		FOIL
$(\_\_\_x^2 - 4x + 15x - \_\_\_)2x = 1085$		Simplify.
$(10x^2 + \_\_\_x - 6)2x = 1085$		Combine like terms.
$\_\_\_\_\_\_\_\_\_\_ = 1085$		Distributive Property

So, the volume of the pool is ____________________.

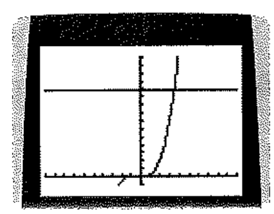
Part B Write and solve a system of equations.

Set each side equal to y to create a system of equations.

$y = 20x^3 + 22x^2 - 12x$ First equation

$y = 1085$ Second equation

Enter the equations in the **Y =** list and graph.



Use the **intersect** feature on the **CALC** menu to find the coordinates of the point of intersection.

The real solution is the x-coordinate of the intersection, which is ____.

Part C Find the dimensions.

Substitute 3.5 feet for x in the length, width, and depth of the pool.

Length: $2x + 3 =$ ____ ft Width: $5x - 2 =$ ____ ft

Depth: $2x =$ ____ ft

Go Online You can complete an Extra Example online.

Think About It!
Is your solution reasonable? Justify your conclusion.