Powers of Binomials

Explore Expanding Binomials

- Online Activity Use interactive tool to complete the Explore.
 - INQUIRY How can you use Pascal's triangle to write expansions of binomials?

Learn Powers of Binomials

You can expand binomials by following a set of rules and using patterns.

Key Concept · Binomial Expansion

In the binomial expansion of $(a + b)^n$,

- there are n + 1 terms.
- n is the exponent of a in the first term and b in the last term.
- in successive terms, the exponent of a decreases by 1, and the exponent of b increases by 1.
- the sum of the exponents in each term is n.
- the coefficients are symmetric.

Pascal's triangle is a triangle of numbers in which a row represents the coefficients of an expanded binomial $(a + b)^n$. Each row begins and ends with 1. Each coefficient can be found by adding the two coefficients above it in the previous row.

Instead of writing out the rows of Pascal's triangle, you can use the Binomial Theorem to expand a binomial. The Binomial Theorem uses combinations to calculate the coefficients of the binomial expansion.

Key Concept · Binomial Theorem

If n is a natural number, then $(a + b)^n =$

$$_{n}C_{0}a^{n}b^{0} + _{n}C_{1}a^{n-1}b^{1} + _{n}C_{2}a^{n-2}b^{2} + _{n}C_{3}a^{n-3}b^{3} + \ldots + _{n}C_{n}a^{0}b^{n}$$

 $1a^{n}b^{0} + \frac{n!}{1!(n-1)!}a^{n-1}b^{1} + \frac{n!}{2!(n-2)!}a^{n-2}b^{2} + \frac{n!}{3!(n-3)!}a^{n-3}b^{3} + \ldots + 1a^{0}b^{n}$

Example 1 Use Pascal's Triangle

Use Pascal's triangle to expand $(x + y)^7$.

Today's Goal

· Expand powers of binomials by using Pascal's Triangle and the Binomial Theorem.

Today's Vocabulary Pascal's triangle

Think About It!

Both $_{n}C_{0}$ and $_{n}C_{n}$ equal 1. What does this mean for the terms of a binomial expansion? How does this relate to Pascal's triangle?

Study Tip

Combinations Recall that C, refers to the number of ways to choose r objects from n distinct objects. In the Binomial Theorem, n is the exponent of $(a+b)^n$, and r is the exponent of b in each term. To calculate the coefficients, remember that n! represents n factorial. This is the product of all counting numbers beginning with n and counting backward to 1. For example, $3! = 3 \cdot 2 \cdot 1$. The value of 0! is defined as 1.

Describe a shortcut vou could use to write out rows of Pascal's triangle instead of adding to find every number in a row. Explain your reasoning.

0	Example	2	Use	the	Binomial	Theorem
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BASEBALL In 2016, the Chicago Cubs won the World Series for the first time in 108 years. During the regular season, the Cubs played the Atlanta Braves 6 times, winning 3 games and losing 3 games. If the Cubs were as likely to win as to lose, find the probability of this outcome by expanding $(w + \ell)^6$.

$$(w + \ell)^6$$

$$= {}_{6}C_{0}w^{6} + {}_{6}C_{1}w^{5}\ell + \underline{\qquad} + {}_{6}C_{3}w^{3}\ell^{3} + {}_{6}C_{4}w^{2}\ell^{4} + \underline{\qquad} + {}_{6}C_{6}\ell^{6}$$

$$= w^{6} + \underline{\qquad} + \frac{6!}{2!4!}w^{4}\ell^{2} + \underline{\qquad} + \frac{6!}{4!2!}w^{2}\ell^{4} + \frac{6!}{5!}w\ell^{5} + \ell^{6}$$

$$= + 6w^{5}\ell + 15w^{4}\ell^{2} + 20w^{3}\ell^{3} + 15w^{2}\ell^{4} + \underline{\qquad} + \ell^{6}$$

By adding the coefficients, you can determine that there were 64 combinations of wins and losses that could have occurred.

represents the number of combinations of 3 wins and

3 losses. Therefore, there was a — or about a _____% chance of the

Cubs winning 3 games and losing 3 games against the Braves.

Study Tip

Assumptions To use the Binomial Theorem, we assumed that the teams had an equal chance of winning and losing. Assuming that allows us to reasonably estimate the probability of an outcome with only the coefficient. To find probabilities of events that are not equally likely, substitute the probability of each event for a and b in the expansion of $(a + b)_{n}$.

Check

GAME SHOW A group of 8 contestants are selected from the audience of a television game show. If there are an equal number of men and women in the audience, find the probability of the contestants being 5 women and 3 men by expanding $(w + m)^8$. Round to the nearest percent if necessary. ____%

Study Tip

Coefficients When the binomial to be expanded has coefficients other than 1, the coefficients will no longer be symmetric. In these cases, it may be easier to use the Binomial Theorem.

Example 3 Coefficients Other Than 1

Expand $(2c - 6d)^4$.

$$(2c - 6d)^4$$

Go Online You can complete an Extra Example online.