# Dividing Polynomials

## Explore Using Algebra Tiles to Divide Polynomials

**Online Activity** Use algebra tiles to complete the Explore.

**@ INQUIRY** How can you use a model to divide polynomials? ×

## Learn Dividing Polynomials by Using Long Division

To divide a polynomial by a monomial, find the quotient of each term of the polynomial and the monomial.

$$\frac{6x^2 - 15x}{3x} = \frac{6x^2}{3x} - \frac{15x}{3x} = 2x - 5$$

You can divide a polynomial by a polynomial with more than one term by using a process similar to long division of real numbers. This process is known as the Division Algorithm. The resulting quotient may be a polynomial or a polynomial with a remainder.

### Key Concept • Division Algorithm

Words: If $f(x)$ and $g(x)$ are two polynomials in which $g(x) \neq 0$ and the degree of $g(x)$ is less than the degree of $f(x)$, then there exists a unique quotient $q(x)$ and a unique remainder $r(x)$ such that $f(x) = q(x)g(x) + r(x)$, where the remainder is zero or the degree of $r(x)$ is less than the degree of $g(x)$.

Symbols: $\frac{f(x)}{g(x)} = q(x) \rightarrow f(x) = q(x)g(x)$

$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \rightarrow f(x) = q(x)g(x) + r(x)$

Example: $\frac{2x^2 - 5x - 3}{x - 3} = 2x + 1 \rightarrow$

$2x^2 - 5x - 3 = (x - 3)(2x + 1)$

$= 2x^2 - 5x - 3$

$\frac{x^2 - 4x - 1}{x + 1} = x - 5 + \frac{4}{x + 1} \rightarrow$

$x^2 - 4x - 1 = (x - 5)(x + 1) + 4$

$= x^2 - 5x - 5 + 4$

$= x^2 - 5x - 1$

**Go Online** You can complete an Extra Example online.

> **Talk About It**
> Is the following statement *always*, *sometimes*, or *never* true? Justify your argument.
>
> If a quadratic polynomial is divided by a binomial with a remainder of 0, the binomial is a factor of the polynomial.

> **Study Tip:**
> In algebra, *unique* means *only one*. So, there is only one quotient and remainder that will satisfy the Division Algorithm for each polynomial.

## Example 1 Divide a Polynomial by a Monomial

**Find $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$.**

$(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$

$$= \frac{24a^4b^3 + 18a^2b^2 - 30ab^3}{\underline{\quad\quad}}$$     Write a fraction.

$$= \frac{24a^4b^3}{6ab} + \frac{18a^2b^2}{6ab} - \frac{30ab^3}{6ab}$$     Sum of Quotients

$$= \frac{24}{6}a^{4-1}b^{3-1} + \frac{18}{6}a^{2-1}b^{2-1} - \frac{30}{6}a^{1-1}b^{3-1}$$     Divide.

$$= \underline{\quad\quad} + 3ab - \underline{\quad\quad}$$     Simplify.

### Check

Find $(9x^9y^5 + 21x^4y^4 - 12x^3y^2) \div (3x^2y^2)$. _____

---

## Example 2 Divide a Polynomial by a Binomial

**Find $(x^2 - 5x - 36) \div (x + 4)$.**

$$
\begin{array}{r}
x - 9 \\
x + 4 \overline{) x^2 - 5x - 36} \\
\underline{(-)\,x^2 + \phantom{0}x} \\
\underline{\quad}x - 36 \\
\underline{(-)-9x - 36}
\end{array}
$$

The quotient is $x - \underline{\quad}$ and the remainder is $\underline{\quad}$.

### Check

Find $\dfrac{x^2 + 6x - 112}{x - 8}$. _____

---

## Example 3 Find a Quotient with a Remainder

**Find $\dfrac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.**

$$
\begin{array}{r}
3z^2 + z - 2 \\
z - 5 \overline{) 3z^3 - 14z^2 - 7z + 3} \\
\underline{(-)3z^3 - \phantom{00}z^2} \\
z^2 - 7z \\
\underline{(-)z^2 \phantom{0000}} \\
-2z + \underline{\quad} \\
\underline{(-)-2z + 10}
\end{array}
$$

🌐 **Go Online** You can complete an Extra Example online.

The quotient is $3z^2 + z - 2$ and the remainder is ____.

Therefore, $\dfrac{3z^3 - 14z^2 - 7z + 3}{} = \underline{\phantom{}}z^2 + z - 2 - \dfrac{}{z - 5}$

## Check

Find the quotient of $(-4x^3 + 5x^2 - 2x - 9)(x - 2)^{-1}$.

_____

---

## Learn Dividing Polynomials by Using Synthetic Division

**Synthetic division** is an alternate method used to divide a polynomial by a binomial of degree 1. You may find this to be a quicker, simpler method.

### Key Concept • Synthetic Division

**Step 1** After writing a polynomial in standard form, write the coefficients of the dividend. If the dividend is missing a term, use 0 as a placeholder. Write the constant $a$ of the divisor $x - a$ in the box. Bring the first coefficient down.

**Step 2** Multiply the number just written in the bottom row by $a$, and write the product under the next coefficient.

**Step 3** Add the product and the coefficient above it.

**Step 4** Repeat **Steps 2** and **3** until you reach a sum in the last column.

**Step 5** Write the quotient. The numbers along the bottom row are the coefficients of the quotient. The power of the first term is one less than the degree of the dividend. The final number is the remainder.

## Example 4 Use Synthetic Division

**Find $(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.**

**Step 1** Write the coefficients of the dividend and write the constant $a$ in the box. Bring the first coefficient, 3, down.
Because $x + 3 = x - (-3)$, $a = -3$.

**Step 2** Multiply by $a$ and write the product.
The product of the coefficient and $a$ is $3 \cdot (-3) = -9$.

**Step 3** Add the product and the coefficient.

**Step 4** Repeat **Steps 2** and **3** until you reach a sum in the last column.

**Step 5** Write the quotient.

$$
\begin{array}{r|rrrr}
\underline{\phantom{-3}} & 3 & -2 & -53 & -60 \\
 & & -9 & 33 & 60 \\
\hline
 & \underline{\phantom{3}} & \underline{\phantom{-2}} & \underline{\phantom{-53}} & \underline{\phantom{-60}}
\end{array}
$$

**Go Online** You can complete an Extra Example online.

> 🧁 **Think About It!**
> What is another method you could use to check your answer?

Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2. The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is _____.

## Check

Find the quotient of $(4x^3 - 8x^2 - 5x + 20) \div (x - 2)$.

_____

---

## Example 5 Divisor with Coefficient Other Than 1

Find $\dfrac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$.

To use synthetic division, the lead coefficient of the divisor must be ___.

$$\dfrac{(4x^4 - 37x^2 + 4x + 9) \div}{(2x - 1) \div} \qquad \text{Divide the numerator and denominator by 2.}$$

$$= \dfrac{2x^4 - x^2 + x + \frac{9}{2}}{x -} \qquad \text{Simplify the numerator and denominator.}$$

$x - a = x - \frac{1}{2}$, so $a = $ _____.

Complete the synthetic division.



The resulting expression is $2x^3 + x^2 - 18x - 7 + \dfrac{1}{x - \frac{1}{2}}$. Simplify the fraction.

$$\dfrac{1}{x - \frac{1}{2}} = \dfrac{(1)2}{\left(x - \frac{1}{2}\right) \cdot 2} \qquad \text{Multiply the numerator and denominator by 2.}$$

$$= \dfrac{\quad}{\quad} \qquad \text{Simplify.}$$

The solution is $2x^3 + x^2 - 18x - 7 + \dfrac{2}{2x - 1}$

You can check your answer by using long division.

## Check

Find $(4x^4 + 3x^3 - 12x^2 - x + 6)(4x + 3)^{-1}$.

_____

🌐 **Go Online** You can complete an Extra Example online.

---

**Watch Out!**

**Missing terms** Add placeholders for terms that are missing from the polynomial. In this case, there are 0 $x^3$-terms.