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Operations with Polynomials

Learn Adding and Subtracting Polynomials

A polynomial is a monomial or the sum of two or more monomials. A binomial is the sum of two monomials, and a trinomial is the sum of three monomials. The degree of a polynomial is the greatest degree of any term in the polynomial.

Polynomials can be added or subtracted by performing the operations indicated and combining like terms. You can subtract a polynomial by adding its additive inverse.

The sum or difference of polynomials will have the same variables and exponents as the original polynomials, but possibly different coefficients. Thus, the sum or difference of two polynomials is also a polynomial.

A set is **closed** if and only if an operation on any two elements of the set produces another element of the same set. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is closed under the operations of addition and subtraction.

Example 1 Identify Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a.
$$x^6 + \sqrt[3]{x} - 4$$

This expression is _____ a polynomial because _____ is not a monomial.

b.
$$5a^4b + 3a^2b^7 - 9$$

This expression _____ a polynomial because each term is a \perp The degree of the first term is 4 + 1 or 5, the degree of the second term is 2 + 7 or 9, and the degree of the third term is 0. So, the degree of the polynomial is _____.

c.
$$\frac{2}{3}x^{-5} - 6x^{-3} - x$$

The expression is _____ a polynomial because x^{-5} and x^{-3} are not

Check

State the degree of each polynomial.

a.
$$x^7 + 6x^5 - \frac{1}{3}$$

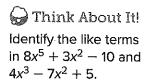
b.
$$3c^7d^2 + 5cd - 9$$

c.
$$p^{10}$$

Go Online You can complete an Extra Example online.

Today's Standards A.APR.1 MP2, MP8

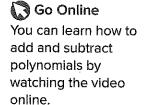
Today's Vocabulary binomial trinomial



Study Tip:

Degree 0 and 1

Remember that constant terms have a degree of 0 and variable term with no exponent indicated has a degree of 1.





Why is it helpful to insert placeholders for the $0x^3$ and $0x^5$ terms? Why were placeholders not included for the x^2 -term in either polynomial?

Study Tip:

Additive Inverse

Distributing -1 to each term in the polynomial being subtracted is the same as finding and adding its additive inverse.

Example 2 Add Polynomials

Find $(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$.

Method 1 Add horizontally.

Group and combine like terms.

$$(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1) = (6x^3 + x^3) + (7x^2 - 4x^2) + (-2x - 8x) + (5 + 1)$$
$$= x^3 + x^2 - x + x$$

Method 2 Add vertically.

Align like terms vertically and add.

$$6x^{3} + 7x^{2} - 2x + 5$$

$$(+)x^{3} - 4x^{2} - 8x + 1$$

$$= x^{3} + x^{2} - x +$$

Check

Find
$$(2x^3 + 9x^2 + 6x - 3) + (4x^3 - 7x^2 + 5x)$$
.

$$\begin{bmatrix} x^3 + \end{bmatrix} x^2 + \begin{bmatrix} x + \end{bmatrix}$$

Example 3 Subtract Polynomials

Find
$$(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$$
.

Method 1 Subtract horizontally.

Group and combine like terms.

$$(2x^{5} + 11x^{4} + 7x - 8) - (5x^{4} + 9x^{3} - 3x + 4) = 2x^{5} + 11x^{4} + 7x - 8 - 5x^{4} - 9x^{3} + 3x - 4$$

$$= 2x^{5} + (11x^{4} - 5x^{4}) + (-9x^{3}) + (7x + 3x) + (-8 + 4)$$

$$= \underline{\qquad} x^{5} + \underline{\qquad} x^{4} - \underline{\qquad} x^{3} + \underline{\qquad} x^{4} - \underline{\qquad} x^{4} + \underline{$$

Method 2 Add vertically.

Align like terms vertically and subtract by adding the additive inverse.

$$2x^{5} + 11x^{4} + 0x^{3} + 7x - 8$$

$$(-)0x^{5} + 5x^{4} + 9x^{3} - 3x + 4$$

$$= x^{5} + x^{4} - x^{3} + x -$$

Check

Find
$$(8x^2 - 3x + 1) - (5x^3 + 2x^2 - 6x - 9)$$
.

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Explore Multiplying Polynomials

- Online Activity Use a table to complete the Explore.
 - INQUIRY How is using a table to multiply polynomials related to the Distributive Property?

Learn Multiplying Polynomials

Polynomials can be multiplied by using the Distributive Property to multiply each term in one polynomial by each term in the other. When polynomials are multiplied, the product is also a polynomial. Therefore, the set of polynomials is closed under the multiplication. This is similar to the system of integers, which is also closed under multiplication. To multiply two binomials, you can use a shortcut called the FOIL method.

Key Concept • FOIL Method

Words: Find the sum of the products of **F** the *First* terms, **O** the *Outer* terms, **I** the *Inner* terms, and **L** the *Iast* terms.

Symbols:

Product of Product of Product of Inner Terms Last Terms

$$(2x + 4)(x - 3) = (2x)(x) + (2x)(-3) + (4)(x) + (4)(-3)$$

$$= 2x^2 - 6x + 4x - 12$$

$$= 2x^2 - 2x - 12$$

Example 4 Simplify by Using the Distributive Property Find $2x(4x^3 + 5x^2 - x - 7)$.

$$2x(4x^3 + 5x^2 - x - 7) = 2x(4x^3) + 2x(\underline{\hspace{1cm}}) + 2x(-x) + 2x(\underline{\hspace{1cm}})$$
$$= \underline{\hspace{1cm}} x^4 + 10x^3 - \underline{\hspace{1cm}} x^2 - 14x$$

Example 5 Multiply Binomials.

Find (3a + 5)(a - 7)(4a + 1).

Step 1 Multiply any two binomials.

$$(3a + 5)(a - 7) = 3a(\underline{\ }) + 3a(\underline{\ }) + 5(a) + 5(-7)$$
 FOIL Method
$$= 3a^2 - 21a + \underline{\ }a - \underline{\ }$$
 Multiply.
$$= 3a^2 - \underline{\ }a - 35$$
 Combine like terms.

(continued on the next page)

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Think About It!

Why are the exponents

multiply the monomials?

added when you

Go Online

for an example of how to multiply two trinomials.

Think About It!

What does x represent in the polynomial expression for the volume of the cake?

Problem-Solving Tip:

Solve a Simpler **Problem Some** complicated problems can be more easily solved by breaking them into several simpler problems. In this case, finding the volume of each tier individually simplifies the situation and makes finding total volume easier.

| | - |
|--|---|
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Step 2 Multiply the result by the remaining binomial.

$$(3a^{2} - 16a - 35)(4a + 1)$$

$$= 3a^{2}(4a + 1) + (\underline{})(4a + 1) + (\underline{})(4a + 1)$$

$$= \underline{} + 3a^{2} - 64a^{2} - 16a - 140a - 35$$

$$= 12a^{3} - \underline{} - 156a \underline{}$$

Check

Find
$$(-2r-3)(5r-1)(r+4)$$
.

| F | | п г | | |
|----|---|----------------------|---|---|
| 3م | + | $\int r^2 + \lfloor$ | r | + |

Example 6 Write and Simplify a Polynomial Expression

BAKING Alo is baking a three-tier cake. Each tier will have $\frac{1}{2}$ the volume of the previous tier. The dimensions of the first tier are 4x - 3, 2x + 1, and x. Find the total volume of the cake.

Step 1 Find the volume of tier 1.

$$V = \text{length} \cdot \text{width} \cdot \text{height} = (4x - 3)(2x + 1)x$$

Simplify the expression by using the Distributive Property.

$$(4x - 3)(2x + 1)x = [4x(2x + 1) - 3(2x + 1)]x$$

$$= (8x^{2} + 4x - 6x - 3)x$$

$$= (8x^{2} - 2x - 3)x$$

$$= 8x^{3} - 2x^{2} - 3x$$

The volume of tier 1 is $\underline{x}^3 - \underline{x}^2 - \underline{x}$.

Step 2 Find the volume of tier 2.

The volume of the second tier is half the volume of tier 1.

$$\frac{1}{2}(8x^3 - 2x^2 - 3x)$$
 or $4x^3 - x^2 -$

Step 3 Find the volume of tier 3.

The volume of the third tier is half the volume of tier 2.

$$\frac{1}{2}(4x^3 - x^2 - 1.5x)$$
 or $x^3 - x^2 - x^2 - x$

Step 4 Find the total volume.

Add the polynomial expressions for the volume of each tier to find the total volume of the cake.

$$(8x^3 - 2x^2 - 3x) + (4x^3 - x^2 - 1.5x) + (2x^3 - 0.5x^2 - 0.75x)$$

The volume of the cake can be represented by:

$$x^3 - x^2 - x$$

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