

## 4.2 Analyzing Graphs of Polynomial Functions • Form A

### Examples 1

- Determine the consecutive integer values of  $x$  between which each real zero of each function is located by using a table.
  - Then sketch the graph on attached graph paper.
  - Use a table to graph each function. Then estimate the  $x$ -coordinates at which relative maxima and relative minima occur.
- $f(x) = -x^3 + 2x^2 - 4$
  - $f(x) = -x^4 - x^3 + 4$

### Examples 2

Use a table to graph each function. Then estimate the  $x$ -coordinates at which relative maxima and relative minima occur.

- $f(x) = 2x^3 - 4x^2 - 3x + 4$
- $f(x) = x^4 + 8x^2 - 12$

### Example 3

- HEIGHT** A plant's height is modeled by the function  $f(x) = 1.5x^3 - 20x^2 + 85x - 84$ , where  $x$  is the number of weeks since the seed was planted and  $f(x)$  is the height of the plant. Graph the function and describe its key features over its relevant domain.

### Example 4

- POPULATION** The table shows the population in Cincinnati, Ohio, since 1960. Make a scatter plot and a curve of best fit to show the trend over the given time period. Then use the equation to estimate the population of Cincinnati in 2020.

Year	Population	Year	Population
1960	502,550	1990	364,553
1970	452,524	2000	331,258
1980	385,457	2010	296,943

### Example 5

- FARMS** The table shows the number of farms in the U.S. at various years, according to the USDA Census of Agriculture. Find the average rate of change from 1982 to 2012. Interpret the results in the context of the situation.

Year	Farms	Year	Farms
1982	2,480,000	2002	2,130,000
1987	2,340,000	2007	2,200,000
1992	2,180,000	2012	2,110,000
1997	2,220,000		

### Mixed Exercises

Graph each function (on attached graph paper) by using a table of values. Then, estimate the  $x$ -coordinates at which each zero and relative extrema occur, and state the domain and range.

- $f(x) = x^3 - 3x^2 + 1$
- $f(x) = 2x^3 - 3x^2 + 2$

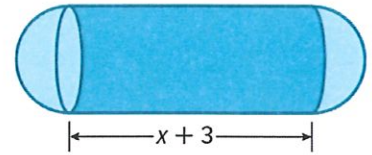
- Determine the key features for  $y = \begin{cases} x^2 & \text{if } x \leq -4 \\ 5 & \text{if } -4 < x \leq 0 \\ x^3 & \text{if } x > 0 \end{cases}$

**USE TOOLS** Use a graphing calculator to estimate the  $x$ -coordinates at which the maxima and minima of each function occur. Round to the nearest hundredth.

11.  $f(x) = -2x^3 + 4x^2 - 5x + 8$

12.  $f(x) = x^5 - 4x^3 + 3x^2 - 8x - 6$

13. **USE TOOLS** A canister has the shape of a cylinder with spherical caps on either end. The volume of the canister in cubic millimeters is modeled by the function  $V(x) = \pi(x^3 + 3x^2) + \frac{4}{3}\pi x^3$  where  $x$  represents the radius in millimeters of a canister that is  $x+3$  millimeters wide.



- Use a graphing calculator to sketch the model that represents the volume of the canister. Include axes labels.
  - What is the domain of the model? Explain any restrictions that apply.
14. **FORECASTING** The table shows the number of deliveries a grocery store has made since they began to offer the service. Use a scatter plot and a curve of best fit to determine the number of deliveries the grocery store can expect to deliver 10 years after they begin the service.

Year	Deliveries	Year	Deliveries
1	60	5	175
2	193	6	156
3	235	7	195
4	210	8	328

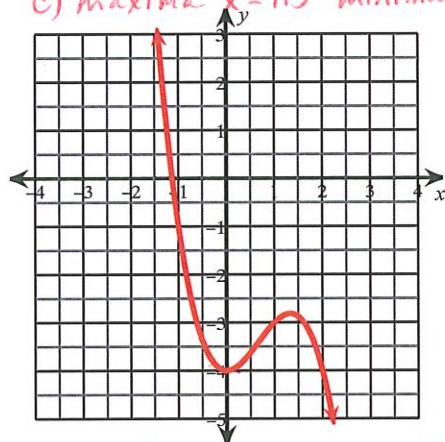
15. **ANALYZE** Explain why the leading coefficient and the degree are the only determining factors in the end behavior of a polynomial function. *The degree & leading coefficient dominate the values of  $x$  as they approach  $+\infty$  &  $-\infty$ .*
16. **CREATE** Sketch the graph of an odd degree polynomial function with 6 extrema and a relative extrema at  $y = 0$ .  
*see sketch.*
17. **PERSEVERE** A function is said to be even if for every  $x$  in the domain of  $f$ ,  $f(x) = f(-x)$ . Is every even-degree polynomial function also an even function? Explain.

*No, it is only even if  $f(-x) = f(x)$ . It must be able to reflect across the  $y$ -axis.*

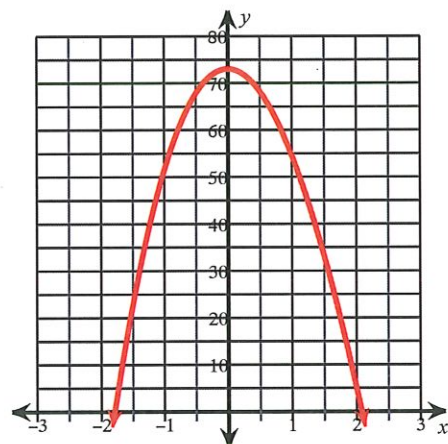
## 4.2 Analyzing Graphs of Polynomial Functions

Name \_\_\_\_\_

- 1) a) zero between  $x = -2$  &  $x = -1$   
 c) maxima  $x = 1.5$  minima  $x = 0$



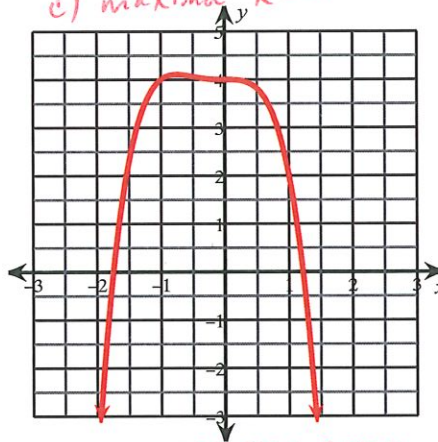
- 3) graph: max:  $x = -0.3$  minima:  $x = 1.6$   
 5)  $D: [0, \infty)$   $R: [0, \infty)$



7)

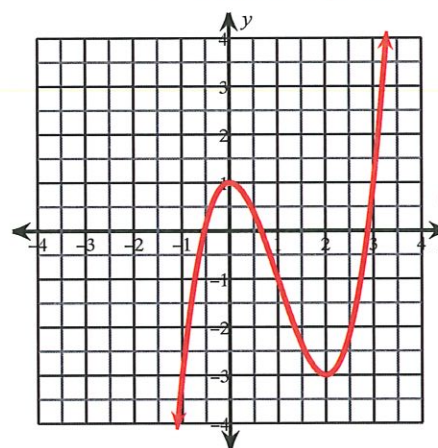
Average rate of change  
 from 1982 to 2012  
 -12333

- 2) a) zeros between  $x = -2$  &  $x = -1$ ,  $x = 1$  &  $x = 2$   
 c) maxima  $x = -1$



- 4) graph  $x$  max: none min  $x = 0$   
 6) ~~X~~

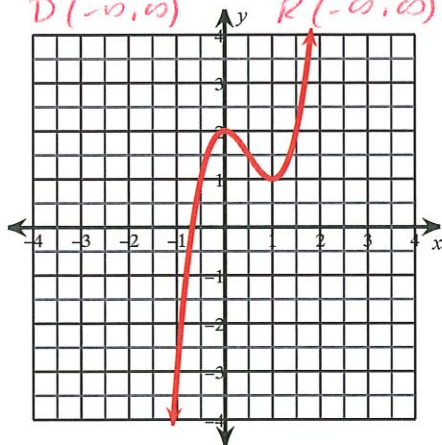
- 8) zeros between  $x = -2$  &  $x = -1$ ,  
 $x = 0$  &  $x = 1$ ,  $x = 1$  &  $x = 2$



max:  $x = -1$   
 min  $x = 1$   
 $D: (-\infty, \infty)$   
 $R: (-\infty, \infty)$

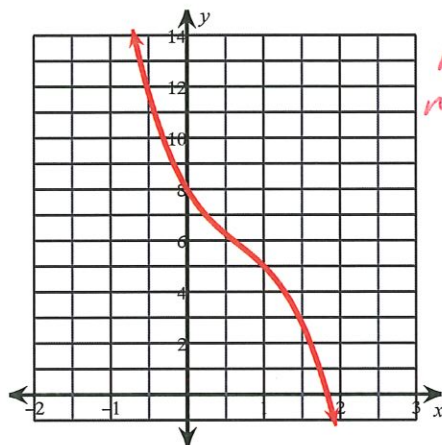


- 9) zero between  $x = -1$  &  $x = 0$   
 max  $x = 0$  min  $x = 1$   
 $D(-\infty, \infty)$   $R(-\infty, \infty)$



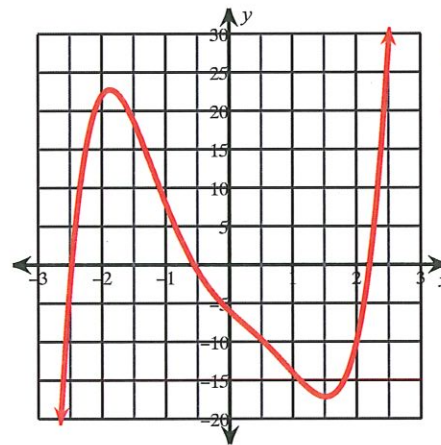
10) X

11)



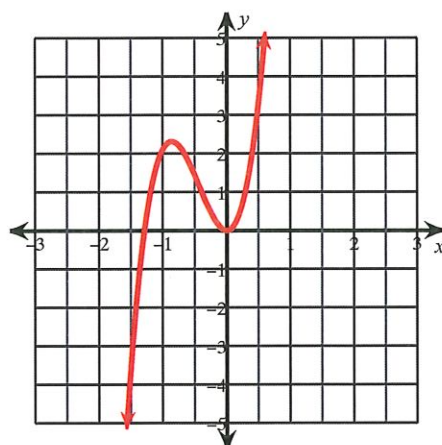
no max  
no min

12)



max  $x \approx -1.87$   
min  $x \approx 1.52$

13)



$D: [0, \infty)$   
 $R: \mathbb{R}$