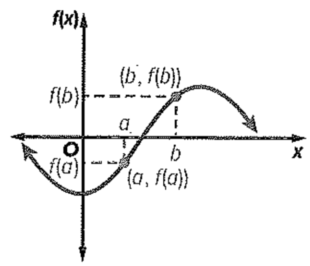
Analyzing Graphs of Polynomial Functions

Learn The Location Principle

If the value of $f(x)$ changes signs from one value of x to the next, then there is a zero between those two x-values. This is called the Location Principle.

Key Concept • Location Principle

Suppose $y = f(x)$ represents a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) < 0$. Then the function has at least one real zero between a and b.



Example 1 Locate Zeros of a Function

Determine the consecutive integer values of x between which each real zero of $f(x) = x^4 - 2x^3 - x^2 + 1$ is located. Then draw the graph.

Step 1 Make a table.

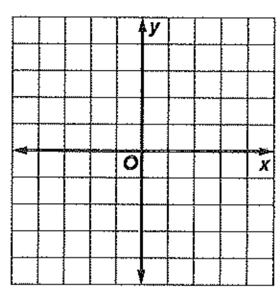
Because $f(x)$ is a fourth-degree polynomial, it will have as many as _____ real zeros or none at all.

x	-2	-1	0	1	2	3	4
$f(x)$		3		-1	-3		

Using the Location Principle, there are zeros between $x = 0$ and $x = 1$ and between $x = 2$ and $x = 3$.

Step 2 Sketch the graph

Use the table to sketch the graph and find the locations of the zeros.



Check

Using the **zero** feature on a graphing calculator, the zeros are located at $x \approx 0.7213$ and $x \approx 2.3486$, which confirms the estimates.

Go Online You can complete an Extra Example online.

Today's Standards
F.IF.4, F.IF.7c
MP4, MP5

Today's Vocabulary
turning point

Think About It!

Not all real zeros can be found by using the Location Principle. Provide an example where $f(a) > 0$ and $f(b) > 0$, but there is a zero between $x = a$ and $x = b$.

Think About It!

How can you adjust the table on your graphing calculator to give a more precise interval for the value of each zero?

Your Notes

Study Tip:

Odd Functions Some odd functions, such as $f(x) = x^3$, have no turning points.

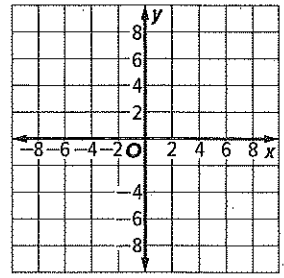
Talk About It

Determine whether the following statement is *sometimes, always,* or *never* true. Justify your reasoning.

For any continuous polynomial function, the y-coordinate of a turning point is also either a relative maximum or relative minimum.

Check

Graph $f(x) = 2x^4 + x^3 - 3x^2 - 2$ and determine the consecutive integer values of x between which each real zero of is located.

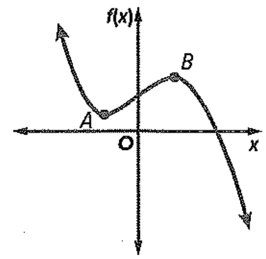


$x =$ _____ and $x =$ _____

$x =$ _____ and $x =$ _____

Learn Extrema and Turning Points

A **turning point** is a change in direction of a graph. The turning points occur at relative maxima or minima of the function.



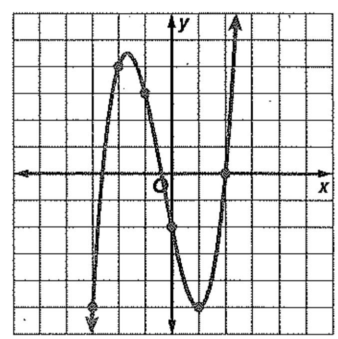
Point A is a relative minimum, and point B is a relative maximum. Both points A and B are turning points. The graph of a polynomial of degree n has at most $n - 1$ turning points.

Example 2 Identify Extrema and Turning Points

Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the x-coordinates at which the relative maxima and relative minima occur.

Step 1 Make a table of values and graph the function.

x	y
−4	−30
−3	
−2	4
−1	3
0	
1	
2	0
3	



Step 2 Estimate the locations of the turning points.

The value of $f(x)$ at $x = -2$ is greater than the surrounding points indicating a turning point *near* $x =$ _____.

The value of $f(x)$ at $x = 1$ is less than the surrounding points indicating a turning point near $x =$ _____.

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Check

Use a table of values of $f(x) = -x^4 - x^3 + 5x^2 + x - 3$ to estimate the x-coordinates at which the relative maxima and relative minima occur.

x	$f(x)$
−3	
−2	
−1	
0	
1	
2	
3	

The relative maxima occur near $x =$ ____ and $x =$ ____.

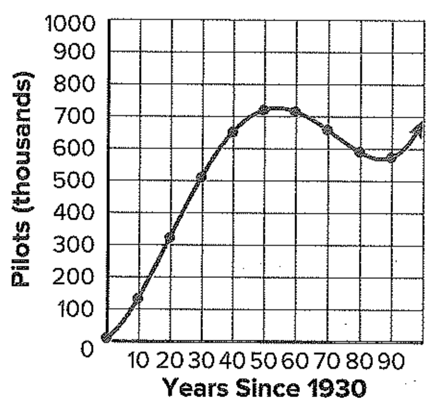
The relative minimum occurs near $x =$ ____.

Example 3 Analyze a Polynomial Function

PILOTS **The total number of certified pilots in the United States is approximated by $f(x) = 0.0000903x^4 - 0.0166x^3 + 0.762x^2 + 6.317x + 7.708$, where x is the number of years after 1930 and $f(x)$ is the number of pilots in thousands. Graph the function and describe its key features over its relevant domain.**

Step 1 Make a table of values and graph the function.

x	y
0	7.708
10	
20	320.496
30	507.961
40	
50	
60	714.616
70	
80	589.356
90	



Step 2 Describe the key features.

Domain and Range

The domain and range of the function is all real numbers. Because the function models years after 1930, the relevant domain and range are $\{x \mid x \geq$ ____$\}$ and $\{f(x) \mid f(x) \geq$ ____$\}$.

(*continued on the next page*)

Study Tip:

Turning Points When graphing with a calculator, keep in mind that a polynomial of degree n has at most $n - 1$ turning points. This will help you to determine whether your viewing window is allowing you to see all of the extrema of the graph.

Go Online

You can learn how to graph and analyze a polynomial function on a graphing calculator by watching the video online.

Think About It!

What trends in the number of pilots does the graph suggest?

Think About It!

It is reasonable that the trend will continue indefinitely? Explain

Study Tip:

Assumptions Determining the end behavior for the graph of a polynomial that models data assumes that the trend continues and there are no other turning points.

Turning Points

There is a relative __________ between 1980 and 1990 and a relative __________ between 2010 and 2020 in the relevant domain.

End Behavior

As $x \to \infty$, $f(x) \to$ ____.

Intercepts

In the relevant domain, the y-intercept is at (0, ______). There is ___ x-intercept, or zero, because the function begins at a value greater than 0 and as As $x \to \infty$, $f(x) \to \infty$.

Symmetry The graph of the function ________ have symmetry.

Check

COINS The number of quarters produced by the United States Mint can be approximated by the function $f(x) = 16.4x^3 - 149.5x^2 - 148.9x + 3215.4$, where x is the number of years since 2005 and $f(x)$ is the total number of quarters produced in millions. Use the graph of the function to complete the table and describe its key features.

Part A Complete the table.

x	Quarters (millions)
0	
2	
4	
6	
8	
10	

Part B Describe the key features.

The relevant domain is ________.

The relevant range is __________________.

There is a relative maximum between ________ and ________.

The y-intercept is ________.

The graph of the function ________ have symmetry.

It is ____________ to assume that the trend will continue indefinitely.

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Example 4 Use Polynomial Functions and Technology to Model

BACKPACKS **The table shows U.S. backpack sales in millions of dollars, according to the Travel Goods Association. Make a scatter plot and a curve of best fit to show the trend over time. Then determine the backpack sales in 2015.**

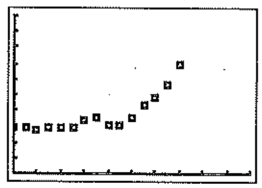
Year	Sales (million $)	Year	Sales (million $)
2000	1140	2008	1246
2001	1144	2009	1235
2002	1113	2010	1419
2003	1134	2011	1773
2004	1164	2012	1930
2005	1180	2013	2255
2006	1364	2014	2779
2007	1436		

Step 1 Enter the data.

Let the year 2000 be represented by 0. Enter the years since 2000 in List 1. Enter the backpack sales in List 2.

Step 2 Graph the scatter plot.

Change the viewing so that all data are visible.



Step 3 Determine the polynomial function of best fit.

```
QuarticReg
 y=ax⁴+bx³+…+e
 a=.1999380264
 b=-3.862237421
 c=26.44767438
 d=-43.6132535
↓e=1139.912195
```

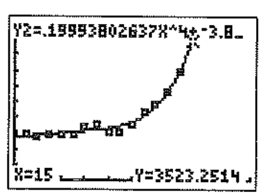
To determine the model that best fits the data, perform linear, quadratic, cubic, and quartic regressions, and compare the coefficients of determination, r^2. The polynomial with a coefficient of determination closest to 1 will fit the data best.

A ______ function fits the data best.

The regression equation with coefficients rounded to the nearest tenths is:

$$y \approx \_\_\_x^4 - \_\_\_x^3 + \_\_\_x^2 - \_\_\_x + \_\_\_.$$

Step 4 Graph and evaluate the regression function.

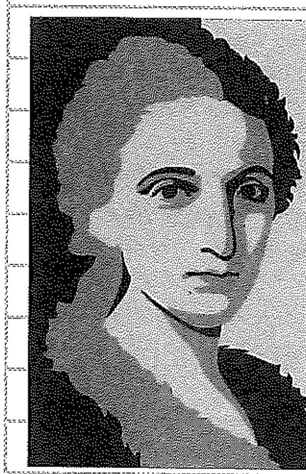


Assuming that the trend continues, the graph of the function can be used to predict backpack sales for a specific year. To determine the total sales in 2015, find the value of the function for $x =$ ____.

In 2015, there were about ______ billion in backpack sales.

Think About It!

Explain the approximation that is made when using the model to determine the backpack sales in a specific year.



Math History Minute:

By the age of 20, Italian mathematician Maria Gaetana Agnesi (1718–1799) had started working on her book *Analytical Institutions,* which was published in 1748. Early chapters included problems on maxima, minima, and turning points. Also described was a cubic curve called the "witch of Agnesi," which was translated incorrectly from the original Italian.

Check

TREES To estimate the amount of lumber that can be harvested from a tree, foresters measure the diameter of each tree. Determine the polynomial function of best fit, where x represents the diameter of a tree in inches and y is the estimated volume measured in board feet. Then estimate the volume of a tree with a diameter of 35 inches.

Diam (in.)	17	19	20	23	25	28	32	38	39	41
Vol (100s of board ft)	19	25	32	57	71	113	123	252	259	294

The polynomial function of best fit is $y =$ ______$x^4 +$ ______$x^3 -$ ______$x^2 +$ ______$x -$ ______.

The estimated volume of a 35-inch diameter tree to the nearest board foot is ______ or ______ of 100s board ft.

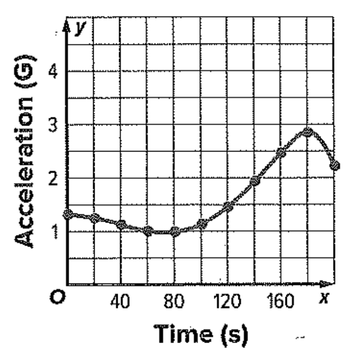
Example 5 Find Average Rate of Change

ROCKETS **The table shows the expected g-force on the Ares-V rocket over the course of its 200-second launch.**

Time (s)	Acceleration (Gs)	Time (s)	Acceleration (Gs)
0	1.34	120	1.46
20	1.26	140	1.93
40	1.12	160	2.47
60	1.01	180	2.84
80	1	200	2.2
100	1.15		

Part A Find the average rate of change.

Estimate: The change in the y-values is about 0.9, and the change in the x-values is 200. So, the rate of change is about $\frac{0.9}{200}$ or ______.



Algebraically:

The average rate of change is $\frac{f(\_\_\_) - f(\_)}{200 - 0}$ or ______.

Part B Interpret the results.

From 0 to ______ seconds, the average rate of change in acceleration was an increase of ______ Gs per ______.

Think About It!

Does the average rate of change from 0 to 200 seconds accurately describe the acceleration 0of the launch? Justify your reasoning.

Go Online You can complete an Extra Example online.