Polynomial Functions

Today's Standards
F.IF.4, F.IF.7c
MP1, MP6

Today's Vocabulary
power function
degree
monomial function
polynomial
standard form of a polynomial
degree of a polynomial
leading coefficient
polynomial function
quartic function
quintic function

Explore Power Functions

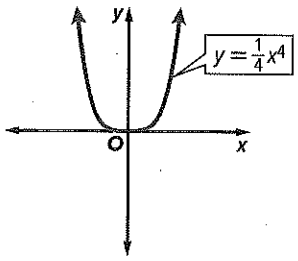
Online Activity Use graphing technology to complete the Explore.

INQUIRY How do the coefficient and degree of a function of the form $f(x) = ax^n$ affect its end behavior?

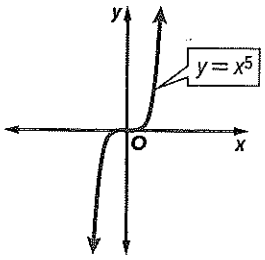
Learn Graphing Power Functions

A **power function** is any function of the form $f(x) = ax^n$ where a and n are nonzero real numbers. For a power function, a is the **leading coefficient** and n is the **degree**, which is the value of the exponent. A power function with positive integer n is called a **monomial function**.

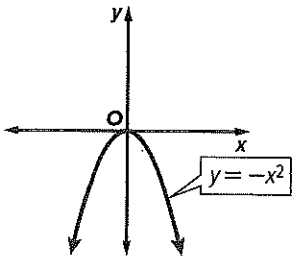
Key Concept • End Behavior of a Monomial Function



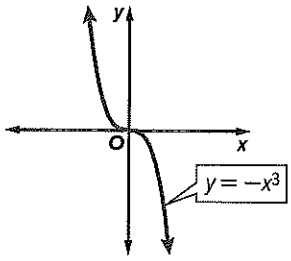
Degree: even
Leading Coefficient: positive
End Behavior:
As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to \infty$
Domain: all real numbers
Range: all real numbers ≥ 0



Degree: odd
Leading Coefficient: positive
End Behavior:
As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to \infty$
Domain: all real numbers
Range: all real numbers



Degree: even
Leading Coefficient: negative
End Behavior:
As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to -\infty$
Domain: all real numbers
Range: all real numbers ≤ 0



Degree: odd
Leading Coefficient: negative
End Behavior:
As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to -\infty$
Domain: all real numbers
Range: all real numbers

Talk About It
Is $f(x) = \sqrt{x}$ power function? a monomial function? Explain your reasoning.

Your Notes

Key Concept • Zeros of Even and Odd Degree Functions

Odd-degree functions will always have at least one real zero. Even-degree functions may have any number of real zeros or no real zeros at all.

Example 1 End Behavior and Degree of Monomial Functions

Describe the end behavior of $f(x) = -2x^3$ using the leading coefficient and degree, and state the domain and range.

leading coefficient: _____, which is _____

degree: _____, which is _____

end behavior: Because the leading coefficient is negative and the degree is odd, the end behavior is that as $x \to -\infty$, $f(x) \to$ _____ and as $x \to \infty$, $f(x) \to$ _____.

domain: _____ range: _____

Check

Describe the end behavior, domain, and range of $f(x) = -10x^6$.

end behavior: As $x \to -\infty$, $f(x) \to$ _____ and as $x \to \infty$, $f(x) \to$ _____.

domain: all real numbers range: all real numbers ≥ 0

Go Online
You can watch a video to see how to graph power functions on a TI-84.

Think About It!
Interpret the domain and range given the context of the situation.

Example 2 Graph a Power Function by Using a Table

PRESSURE **The pressure P given the flow rate F is defined by $P(F) = \frac{3}{2}F^2$. Graph the function $P(F)$, and state the domain and range.**

Step 1 Find a and n.

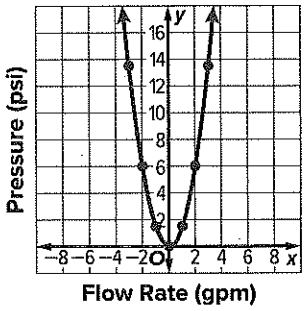
For $P(F) = \frac{3}{2}F^2$, $a =$ _____, and $n =$ _____.

Step 2 State the domain and range.

Because a is positive and n is even, the domain is _____ and the range is all real numbers _____.

Steps 3–5 Create a table of values and graph the ordered pairs.

-2	$\frac{3}{2}(-2)^2$	
-1	$\frac{3}{2}(-1)^2$	1.5
0	$\frac{3}{2}(-0)^2$	
1	$\frac{3}{2}(1)^2$	
2	$\frac{3}{2}(2)^2$	6



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Explore Polynomial Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY How is the degree of a function related to the number of times its graph intersects the x-axis?

Go Online
You can learn how to graph a polynomial function by watching the video online.

Learn Graphing Polynomial Functions

A **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1}, a_1, and a_0 are real numbers, and n is a nonnegative integer. Because the terms are written in order from greatest to least degree, this polynomial is written in **standard form**. The **degree of the polynomial** is n and the leading coefficient is a_n.

A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable. You have learned about constant, linear, quadratic, and cubic functions. A **quartic function** is a fourth-degree function. A **quintic function** is a fifth-degree function. The general shapes of the graphs of several polynomial functions show the maximum number of times the graphs of each function may intersect the x-axis. The degree tells you the maximum number of times that the graph of a polynomial function intersects the x-axis.

Think About It!
If a polynomial function has a leading coefficient of 4, can you determine its end behavior? Explain your reasoning.

Example 3 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $2x^4 - 3x^3 - 4x^2 - 5x + 6a$ degree: ___ leading coefficient: ___

b. $7x^3 - 2$ degree: ___ leading coefficient: ___

c. $4x^2 - 2xy + 8y^2$ This is not a polynomial in one variable. There are two variables, x and y.

d. $x^5 + 12x^4 - 3x^3 + 2x^2 + 8x + 4$ degree: ___ leading coefficient: ___

Think About It!
Jamison says the leading coefficient of $4x^2 - 3 + 2x^3 - x$ is 4. Do you agree or disagree? Justify your reasoning.

Check

Select the degree and leading coefficient of $11x^3 + 5x^2 - 7x - \frac{6}{x}$. ___

A. degree: 3, leading coefficient: 11

B. degree: 11, leading coefficient: 3

C. This is not a polynomial in one variable. There are two variables, x and y.

D. This is not a polynomial in one variable. The term $\frac{6}{x}$ has the variable with an exponent less than 0.

Watch Out!
Leading Coefficients
If the term with the greatest degree has no coefficient shown, as in part d, the leading coefficient is 1.

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Think About It!

What values of x make sense in the context of the situation? Justify your reasoning.

Study Tip:

Labels Notice that the x-axis is measuring the percent of the radius, not the actual length of the radius.

Example 4 Evaluate and Graph a Polynomial Function

SUN **The density of the Sun, in grams per centimeter cubed, expressed as a percent of the distance from the core of the Sun to its surface can be modeled by the function $f(x) = 519x^4 - 1630x^3 + 1844x^2 + 155$, where x represents the percent as a decimal. At the core, $x = 0$, and at the surface $x = 1$.**

Part A Find the core density of the Sun at a radius 60% of the way to the surface.

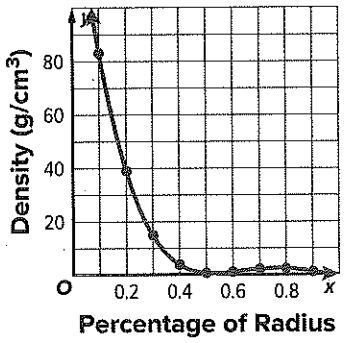
Because we need to find the core density at a radius 60% of the way to the surface, $x = 0.6$. So, replace x with 0.6 and simplify.

$$f(x) = 519x^4 - 1630x^3 + 1844x^2 + 155$$
$$= 519(\_\_\_)^4 - 1630(\_\_\_)^3 + 1844(\_\_\_)^2 + 155$$
$$= 67.2624 - 352.08 + 663.84 - 533.4 + 155$$
$$= \_\_\_\_\_ \frac{g}{cm^3}$$

Part B Sketch a graph of the function.

Substitute values of x to create a table of values. Then plot the points, and connect them with a smooth curve.

x	$f(x)$
0.1	82.9619
0.2	
0.3	
0.4	3.4064
0.5	
0.7	1.7819
0.8	
0.9	



Check

CARDIOLOGY To help predict heart attacks, doctors can inject a concentration of dye in a vein near the heart to measure the cardiac output in patients. In a normal heart, the change in the concentration of dye can be modeled by $f(x) = -0.006x^4 + 0.140x^3 - 0.053x^2 + 1.79x$, where x is the time in seconds.

Part A Find the concentration of dye after 5 seconds.

$f(5) = \_\_\_\_\_$

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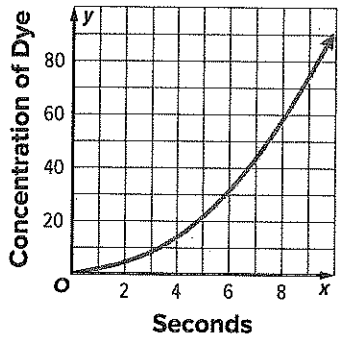
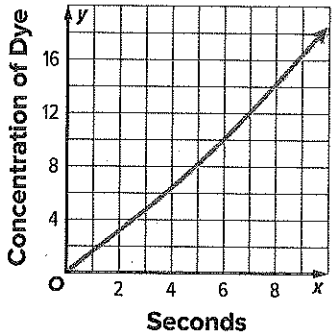
Part B Select the graph of the concentration of dye over 10 seconds. ____

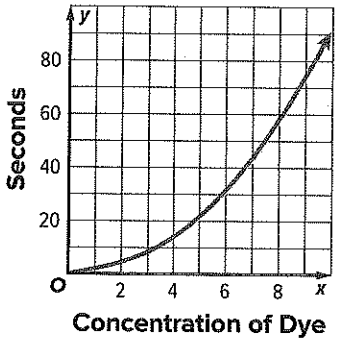
A.



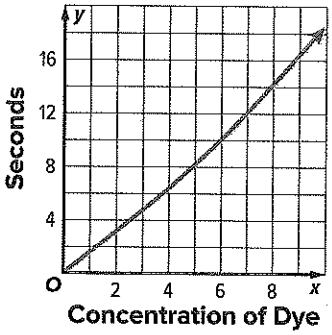
B.



C.

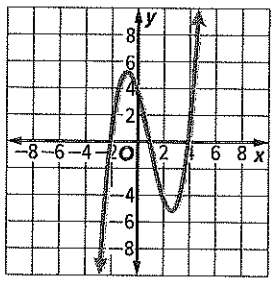


D.



Example 5 Zeros of a Polynomial Function

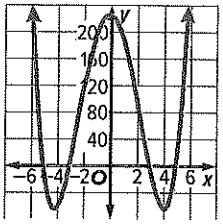
Use the graph to state the number of real zeros of the function.



The real zeros occur at $x = -2$, 1, and 4, so there are _____ real zeros.

Check

Use the graph to state the number of real zeros of the function.



The function has _____ real zero(s).

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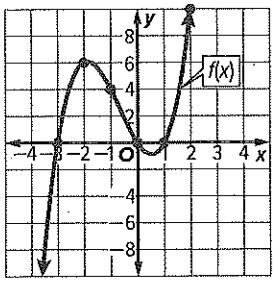
Study Tip:

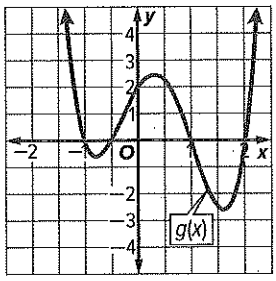
Zeros The real zeros occur at values of x where $f(x) = 0$, or where the function intersects the x-axis. Recall that odd functions have at least one real zero and even functions have any number of real zeros. So, the minimum number of times that an odd function intersects the x-axis is 1, and the minimum number of times that an even function intersects the x-axis is 0.

Example 6 Compare Polynomial Functions

Examine $f(x) = x^3 + 2x^2 - 3x$ and $g(x)$ shown in the graphs.

Part A Graph $f(x)$.





Part B Which function has the greater relative maximum?

$f(x)$ has a relative maximum at $y = 6$, and $g(x)$ has a relative maximum between $y = 2$ and $y = 3$. So, _____ has the greater relative maximum.

Part C Compare the zeros, x- and y-intercepts, and end behavior of $f(x)$ and $g(x)$.

zeros:

$f(x)$: _____, _____, _____

$g(x)$: The graph appears to intersect the x-axis at _____, _____, _____, _____ intercepts:

$f(x)$: x-intercepts: _____, _____, _____; y-intercept: 0

$g(x)$: x-intercepts: _____, _____, _____, _____; y-intercept: 2

end behavior:

$f(x)$: As $x \to -\infty$, $f(x) \to$ _____, and as $x \to \infty$, $f(x) \to$ _____

$g(x)$: As $x \to -\infty$, $g(x) \to$ _____, and as $x \to \infty$, $g(x) \to$ _____

Think About It!

Find the domain and range of $f(x)$. Does $g(x)$ have the same domain and range? Explain.

Study Tip:

Zeros The zeros of a polynomial function are the x-coordinates of the points at which the graph intersects the x-axis.

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

Record your observations here

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