

## 4.1 Polynomial Functions • Form A

### Example 1

Describe the end behavior of each function using the leading coefficient and degree, and state the domain and range.

1.  $f(x) = -2x^3$

EB:  $x \rightarrow \infty, f(x) \rightarrow -\infty$ ;  $x \rightarrow -\infty, f(x) \rightarrow \infty$

LC:  $-2$  Deg:  $3$

D:  $(-\infty, \infty)$  R:  $(-\infty, \infty)$

2.  $f(x) = \frac{3}{4}x^6$

EB:  $x \rightarrow \infty, f(x) \rightarrow \infty$ ;  $x \rightarrow -\infty, f(x) \rightarrow \infty$

LC:  $\frac{3}{4}$  Deg:  $6$

D:  $(-\infty, \infty)$  R:  $[0, \infty)$

### Example 2

3. **MACHINE EFFICIENCY** company uses the function  $f(x) = x^3 + 3x^2 - 18x - 40$  to model the change in efficiency of a machine based on its position  $x$ . Graph the function (on attached graph paper) and state the domain and range.

D:  $(-\infty, \infty)$  R:  $(-\infty, \infty)$

### Example 3

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

4.  $(2x - 1)(4x^2 + 3)$  D:  $3$ ; LC:  $8$

5.  $18 - 3y + 5y^2 - y^5 + 7y^6$  D:  $6$ ; LC:  $7$

6.  $2r - r^2 + \frac{1}{r^2}$  not a poly. in one variable.  
negative exponent (variable in denominator)

### Example 4

7. **DRILLING** The volume of a drill bit can be estimated by the formula for a cone,  $V = \frac{1}{3}\pi hr^2$ , where  $h$

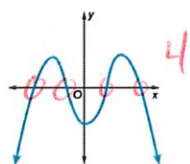
is the height of the bit and  $r$  is its radius. Substituting  $\frac{\sqrt{3}}{3}r$  for  $h$ , the volume of the drill bit can be estimated by  $V = \frac{\sqrt{3}}{9}\pi r^3$ .

- a. What is the volume of a drill bit with a radius of 3 centimeters?  $3\pi\sqrt{3}$   
b. Sketch a graph (on attached graph paper) of the function.

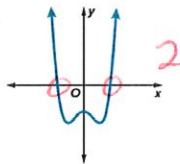
### Example 5

Use the graph to state the number of real zeros of the function.

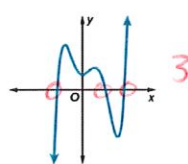
8.



9.

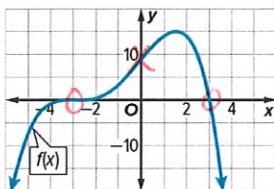


10.



### Example 6

11. Examine the graph of  $f(x)$  and  $g(x)$  shown in the table.



$x$	-5	-3	0	1.5	3
$g(x)$	7.5	0	-9	-15	0

- a. Which function has the greater relative maximum?  $f(x)$   
b. Compare the zeros, x- and y-intercepts, and end behavior of  $f(x)$  and  $g(x)$ .

zeros:  $f(x) = -3, 3$   $g(x) = -3, 3$

y-int:  $f(x) = 9$ ,  $g(x) = -9$

end behavior.

$f(x)$ :  $x \rightarrow \infty, f(x) \rightarrow -\infty$   
 $x \rightarrow -\infty, f(x) \rightarrow -\infty$

$g(x)$ :  $x \rightarrow \infty, g(x) \rightarrow \infty$   
 $x \rightarrow -\infty, g(x) \rightarrow \infty$

**Mixed Exercises**

Describe the end behavior, state the degree and leading coefficient of each polynomial. If the function is not a polynomial, explain why.

12.  $g(x) = 2x^5 + 6x^4$

EB:  $x \rightarrow \infty, g(x) \rightarrow \infty; x \rightarrow -\infty, g(x) \rightarrow \infty$ 

LC: 2 Deg: 5

13.  $h(x) = 9x^6 - 5x^7 + 3x^2$

EB:  $x \rightarrow \infty, h(x) \rightarrow \infty; x \rightarrow -\infty, h(x) \rightarrow \infty$ 

LC: 9 Deg: 6

14.  $f(x) = (5 - 2x)(4 + 3x)$

EB:  $x \rightarrow \infty, f(x) \rightarrow -\infty; x \rightarrow -\infty, f(x) \rightarrow -\infty$ 

LC: -6 Deg: 2

15.  $g(x) = 3x^7 - 4x^4 + \frac{3}{x}$

EB: not a poly in one variable

LC: Deg:

16. **CONSTRUCT ARGUMENTS** Explain why a polynomial function with an odd degree must have at least one real zero.

end behavior on one end goes to  $\infty$  & on the other goes to  $-\infty$ , so it must cross x-axis at least once.

17. **COMPARING** Compare the end behavior of the functions  $g(x) = -3x^4 + 15x^3 - 12x^2 + 3x + 20$  and  $h(x) = -3x^4 - 16x - 1$ . Explain your reasoning.

same degree is even & leading coefficient is negative.

18. **ANALYZE** Compare the functions  $g(x)$  and  $f(x)$ . Determine which function has the potential for more real zeros and the degree of each function.

$g(x) = x^4 + x^3 - 13x^2 + x + 4$
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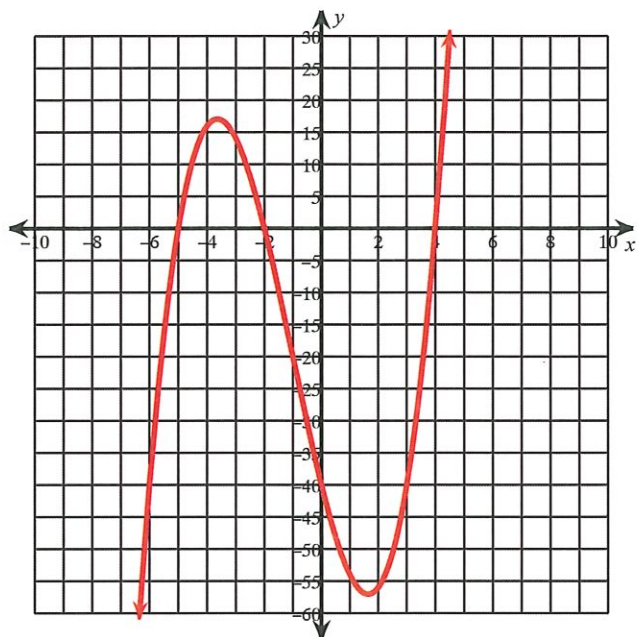
x	-24	-18	-12	-6	0	6	12	18	24
f(x)	-8	-1	3	-2	4	7	-1	-8	5

19. **CREATE** Sketch the graph of an even-degree polynomial with 7 real zeros, one of which is a double zero, and the leading coefficient is negative.

# 4.1 Polynomial Functions - Graphs

Name \_\_\_\_\_

3)



4)

5)

6)

7)

