Solve Quadratic Equations by Factoring

Explore Finding the Solutions of Quadratic Equations by Factoring

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use factoring to solve a quadratic equation?

The **factored form** of a quadratic equation is $0 = a(x - p)(x - q)$, where $a \neq 0$. In this equation, p and q represent the x-intercepts of the graph of the related function. For example, $0 = x^2 - 2x - 3$ can be written in the factored form $0 = (x - 3)(x + 1)$ and its related graph has x-intercepts of -1 and 3.

Learn Solving Quadratic Equations by Factoring

Key Concept • Factoring by Using the Distributive Property

Symbols: $ax + bx = x(a + b)$

Example: $20 + 15 = 5(4 + 3)$

Key Concept • Factoring Trinomials

Symbols: $x^2 + bx + c = (x + m)(x + p)$ when $m + p = b$ and $mp = c$

Example: $x^2 - 8x + 15 = (x - 5)(x - 3)$, because $-5 + (-3) = -8$ and $-5(-3) = 15$

Key Concept • Zero Product Property

Words: For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and $b = 0$.

Example: If $(x - 2)(x + 4) = 0$, then $x - 2 = 0$, $x + 4 = 0$, or $x - 2 = 0$ and $x + 4 = 0$.

Example 1 Use the Distributive Property

Solve $12x^2 - 2x = x$ by factoring. Check your solution.

$12x^2 - 2x = x$	Original equation
$12x^2 - \_\_\_x = 0$	Subtract x from each side.
$3x(\_\_\_) - 3x(\_\_\_) = 0$	Factor the GCF.
$\_\_\_(4x - 1) = 0$	Distributive Property
$3x = 0$ and $4x - 1 = 0$	Zero Product Property.
$x = \_\_\_$ and $x = \_\_\_$	Solve.

Go Online You can complete an Extra Example online.

Today's Standards
N.CN.7, N.CN.8, F.IF.8a
MP1, MP7

Today's Vocabulary
factored form
difference of squares
perfect square trinomials

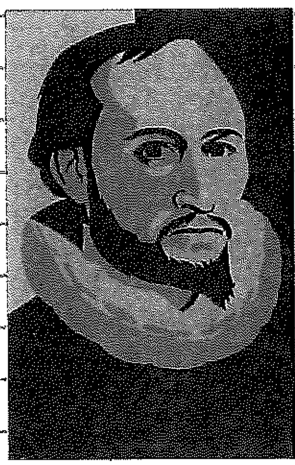
Think About It!
The equation $x^2 - 2x - 3 = 0$ could be solved by factoring, where $x^2 - 2x - 3 = (x - 3)(x + 1)$. How are the factors of the equation related to the roots, or zeros, of the related function $f(x) = x^2 - 2x - 3$?

Go Online You can watch a video to see how to use algebra tiles to factor a polynomial using the Distributive Property online.

Your Notes

Think About It!

Choose two integers and write an equation in standard form with these roots. How would the equation change if the signs of the two roots were switched?



Math History Minute

English mathematician and astronomer **Thomas Harriot (1560–1621)** was one of the first, if not the first, to consider the imaginary roots of equations. Harriot advanced the notation system for algebra and studied negative and imaginary numbers.

Example 2 Factor a Trinomial

Solve $x^2 + 4x - 46 = 71$ by factoring. Check your solution.

$x^2 + 4x - 46 = 71$	Original equation
$x^2 + 4x - \underline{\quad\quad} = 0$	Subtract 71 from each side.
$(x + \underline{\quad\quad})(x - \underline{\quad\quad}) = 0$	Factor the trinomial.
$x + 13 = 0$ or $x - 9 = 0$	Zero Product Property
$x = \underline{\quad\quad}$ $\quad x = \underline{\quad\quad}$	Solve.

Example 3 Solve an Equation by Factoring

ACCELERATION **The equation $d = vt + \frac{1}{2}at^2$ represents the displacement d of a car traveling at an initial velocity v where the acceleration a is constant over a given time t. Find how long it takes a car to accelerate from 30 mph to 45 mph if the car moved 605 feet and accelerated slowly at a rate of 2 feet per second squared.**

Understand

What do you know?

$d = \underline{\quad\quad}$ ft, $v = \underline{\quad\quad}$ mph, and $a = \underline{\quad\quad}$ ft/v^2.

What do you need to find? ____

Plan and Solve

Step 1 Convert so that the units are the same.

$$v = 30\,\frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \times \frac{\underline{\quad\quad}\ \text{ft}}{\cancel{\text{mi}}} \times \frac{1\ \cancel{\text{hr}}}{\underline{\quad\quad}\ \text{s}} = \underline{\quad\quad}\ \frac{\text{ft}}{\text{s}}$$

Step 2 Substitute the known values in the equation.

$d = vt + \frac{1}{2}at^2$	Original equation
$\underline{\quad\quad} = \underline{\quad\quad} + \frac{1}{2}(\underline{\quad\quad})t^2$	$d = 605$, $v = 44$, and $a = 2$

Step 3 Solve the equation for t.

$605 = 44t + \frac{1}{2}(2)t^2$	Equation for displacement
$0 = t^2 - 44t - 605$	Subtract 605 from each side.
$0 = (t + \underline{\quad\quad})(t - \underline{\quad\quad})$	Factor.
$0 = t + 55$ or $0 = t - 11$	Zero Product Property
$t = -55$ $t = 11$	Solve.

Step 4 Interpret answers in the context of the situation.

Because time cannot be negative, $t = \underline{\quad\quad}$ is the only viable solution.

So, it took the car ____ seconds to accelerate to 45 mph.

Go Online You can complete an Extra Example online.

Check

SALES A clothing store is analyzing their market to determine the profitability of their new dress design. If $P(x) = -16x^2 + 1712x - 44{,}640$ represents the store's profit when x is the price of each dress, find the price range the store should charge to make the dress profitable. ____

A. between \$11.25 and \$15.50

B. between \$45 and \$62

C. between \$50 and \$54

D. between \$180 and \$248

Example 4 Factor a Trinomial Where a is Not 1

Solve $3x^2 + 5x + 15 = 17$ by factoring. Check your solution.

$3x^2 + 5x + 15 = 17$	Original equation
$3x^2 + 5x - 2 = 0$	Subtract 17 from each side.
$(3x - 1)(x + 2) = 0$	Factor the trinomial.
$(3x - 1) = 0$ or $x + 2 = 0$	Zero Product Property
$x = \frac{1}{3}$ $x = -2$	Solve.

Check

Solve $4x^2 + 12x - 27 = 13$ by factoring. Check your solution.

$x =$ ____, $x =$ ____.

Think About It!

Explain how to determine which values should be chose for m and p when factoring a polynomial of the form $ax^2 + bx + c$.

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Key Concept • Factoring Differences of Squares

Words: To factor $a^2 - b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Example: $a^2 - b^2 = (a + b)(a - b)$

Key Concept • Factoring Perfect Squares

Words: To factor $a^2 + 2ab + b^2$, find the square roots of a^2 and b^2. Then apply the pattern.

Example: $a^2 + 2ab + b^2 = (a + b)^2$

Not all quadratic equations have solutions that are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$. For example, you know that the solution of $x^2 = 4$ must be complex because there is no real number for which its square is -4. If you take the square root of each side, $x = 2i$ and $-2i$.

Talk About It

Is $5 + 3i$ in simplest form? Explain your reasoning.

Example 5 Factor a Difference of Squares

Solve $81 = x^2$ by factoring. Check your solution.

$81 = x^2$	Original equation
$81 - x^2 = 0$	Subtract x^2 from each side.
$\_\_\_^2 - \_\_\_^2 = 0$	Write in the form $a^2 - b^2$.
$(\_\_ + \_\_)(\_\_ - \_\_) = 0$	Factor the difference of squares.
$9 + x = 0$ or $9 - x = 0$	Zero Product Property
$x = \_\_\_ \quad x = \_\_\_$	Solve.

Check

Solve $x^2 = 529$ by factoring. Check your solution.

$x = \_\_\_, \_\_\_$

Example 6 Factor a Perfect Square Trinomial

Solve $16y^2 - 22y + 23 = 26y - 13$ by factoring. Check your solution.

$16y^2 - 22y + 23 = 26y - 13$	Original equation
$16y^2 - \_\_y + 23 = -13$	Subtract $26y$ from each side.
$16y^2 - 48y + \_\_ = 0$	Add 13 to each side.
$(\_\_)^2 - 2(\_\_)(\_\_) + \_\_^2 = 0$	Factor the perfect square trinomial.
$(\_\_ - \_\_)^2 = 0$	Simplify.
$y = \_\_\_$	Take the square root of each side and solve.

Check

Solve $16x^2 - 22x + 15 = 10x - 1$ by factoring. Check your solution.

$x = \_\_\_$

Example 7 Complex Solutions

Solve $x^2 = -144$ by factoring. Check your solution.

$x^2 = -144$	Original equation
$x^2 + 144 = 0$	Add 144 to each side.
$x^2 - (\_\_\_^2) = 0$	$144 = -(12^2)$
$(x + \_\_)(x - \_\_) = 0$	Factor the difference of squares.
$x + 12i = 0$ or $x - 12i = 0$	Zero Product Property
$x = \_\_\_ \quad x = \_\_\_$	Solve.

Go Online You can complete an Extra Example online.

Go Online You can watch a video to see how to use algebra tiles to factor a difference of squares online.

Think About It!
Why does this equation have one solution instead of two?

Think About It!
Explain why both $(-12i)^2$ and $(12i)^2$ equal -144.

Watch Out!
Complex Numbers Remember i^2 equals -1, not 1.