Complex Numbers

Explore Factoring Prime Polynomials

🔘 **Online Activity** Use guiding exercises to complete the Explore.

> @ **INQUIRY** Can you factor a prime polynomial?

Learn Pure Imaginary Numbers

In your math studies so far, you have worked with real numbers. However, some equations such as $y = x^2 + x + 1$ do not have real solutions. This led mathematicians to define imaginary numbers. The **imaginary unit i** is the principal square root of -1. Thus, $i = \sqrt{-1}$ and $i^2 = -1$.

Numbers of the form $6i$, $-2i$, and $i\sqrt{3}$ are called pure imaginary numbers. A **pure imaginary number** is a number of the form bi, where b is a real number and $i = \sqrt{-1}$ For any positive real number $\sqrt{-b^2} = \sqrt{b^2}\sqrt{-1}$ or bi.

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers. The first few powers of i are shown.

$$i^1 = i \qquad i^2 = -1 \qquad i^3 = i^2 \cdot i \text{ or } -i$$

Example 1 Square Roots of Negative Numbers

Simplify $\sqrt{-294}$.

$$\sqrt{-294} = \sqrt{-1 \cdot 7^2 \cdot 6} \qquad \text{Prime Factorization}$$
$$= \sqrt{}\sqrt{7^2}\sqrt{} \qquad \text{Factor out the imaginary unit.}$$
$$= i \cdot \underline{} \cdot \sqrt{6} \text{ or } 7i\underline{} \qquad \text{Simplify.}$$

Check

Simplify $\sqrt{-75}$. ____

A $i\sqrt{75}$

B $3i\sqrt{5}$

C $5i\sqrt{3}$

D $-3\sqrt{5}$

🔘 **Go Online** You can complete an Extra Example online.

Today's Vocabulary
imaginary unit i
pure imaginary number
complex number
complex conjugates
rationalizing the denominator

🔘 **Go Online**
You may want to complete the Concept Check to check your understanding.

Study Tip:

Square Factors When factoring an expression under a radical, look for perfect square factors.

Example 2 Products of Pure Imaginary Numbers

Simplify $\sqrt{-10}\ \sqrt{-15}$.

$$\sqrt{-10}\ \sqrt{-15} = i\sqrt{10}\ i\sqrt{15} \qquad i = \sqrt{-1}$$
$$= i^2 \cdot \sqrt{150} \qquad \text{Multiply.}$$
$$= -1 \cdot \sqrt{25} \cdot \sqrt{6} \qquad \text{Simplify.}$$
$$= \underline{\quad}\sqrt{6} \qquad \text{Multiply.}$$

Check

Simplify $\sqrt{-16} \cdot \sqrt{-25}$.

____

Example 3 Equation with Pure Imaginary Solutions

Solve $x^2 + 81 = 0$.

$$x^2 + 81 = 0 \qquad \text{Original equation}$$
$$x^2 = \underline{\quad} \qquad \text{Subtract 81 from each side.}$$
$$x = \pm \underline{\quad} \qquad \text{Square Root Property}$$
$$x = \pm \underline{\quad} \qquad \text{Simplify.}$$

ALTERNATE METHOD

$$x^2 + 81 = 0 \qquad \text{Original equation}$$
$$x^2 + \underline{\quad}^2 = 0 \qquad 81 = 9^2$$
$$x^2 - (-\underline{\quad})^2 = 0 \qquad \text{Rewrite in the difference of squares pattern.}$$
$$(x + \underline{\quad})(x - \underline{\quad}) = 0 \qquad \text{Difference of squares:}\ \sqrt{-9^2} = \sqrt{-81} = 9i$$
$$(x + \underline{\quad}) = 0 \text{ or } (x - \underline{\quad}) = 0 \qquad \text{Zero Product Property}$$
$$x = -\underline{\quad} \text{ or } x = \underline{\quad} \qquad \text{Simplify.}$$

Think About It!

Compare and contrast the methods.

Check

Solve $3x^2 + 27 = 0$.

$x = \underline{\quad}$ and $x = \underline{\quad}$

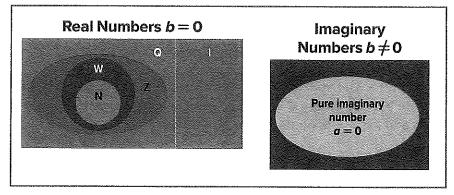 Go Online You can complete an Extra Example online.

Learn Complex Numbers

Key Concept • Complex Numbers

A **complex number** is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part. One example of a complex number is $5 + 2i$. Another example is $1 - 3i$, because it can be written as $1 + -3i$.

The Venn Diagram shows the set of complex numbers. Notice that all of the real numbers are part of the set of complex numbers.

Complex Numbers $(a + bi)$

The Commutative and Associative Properties of Multiplication and Addition and the Distributive Property hold true for complex numbers. To add or subtract complex numbers, combine like terms. That is, combine the real parts, and combine the imaginary parts.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number.

A radical expression is in simplest form if no radicands contain fractions and no radicals appear in the denominator of a fraction. Similarly, a complex number is in simplest form if no imaginary numbers appear in the denominator of a fraction. You can use complex conjugates to simplify a fraction with a complex number in the denominator. This process is called **rationalizing the denominator**.

Example 4 Equate Complex Numbers

Find the values of x and y that make $5x - 7 + (y + 4)i = 13 + 11i$ true.

Use equations relating the real and imaginary parts to solve for x and y.

$5x - 7 = \underline{\quad}$ Real parts

$5x = \underline{\quad}$ Add 13 to each side.

$x = \underline{\quad}$ Divide each side by 5.

$y + 4 = \underline{\quad}$ Real parts

$y = \underline{\quad}$ Add 13 to each side.

Go Online You can complete an Extra Example online.

Think About It!
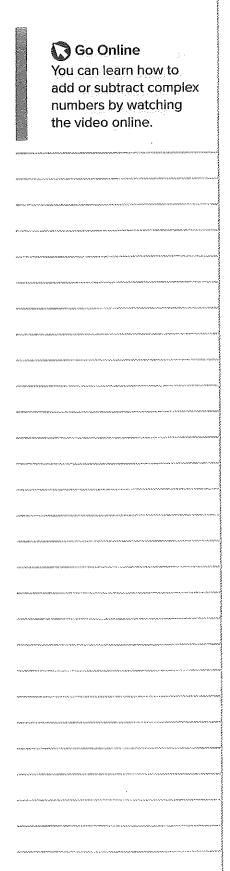
Compare and contrast the subsets of the complex number system using the Venn diagram above.

Example 5 Add or Subtract Complex Numbers

Simplify $(8 + 3i) - (4 - 10i)$.

$(8 + 3i) - (4 - 10i) = (8 - \underline{\quad}) + [3 - (\underline{\quad})]i$ Commutative and Associative Properties

$\qquad\qquad\qquad = \underline{\quad} + 13i$ Simplify.

Check

Simplify $(-5 + 5i) - (-3 + 8i)$.

$\underline{\quad} + \underline{\quad}i$

Example 6 Multiply Complex Numbers

ELECTRICITY **The voltage V of an AC circuit can be found using the formula $V = CI$, where C is current and I is impedance. If $C = 3 + 2j$ amps and $I = 7 - 5j$ ohms, determine the voltage.**

$V = CI$ Voltage Formula

$= (3 + 2j)(7 - 5j)$ $C = 3 + 2j$ and $I = 7 - 5j$

$= 3(\underline{\quad}) + 3(\underline{\quad}) + 2j(\underline{\quad}) = 2j(\underline{\quad})$ FOIL Method

$= \underline{\quad} - 15j + \underline{\quad} - 10j^2$ Multiply.

$= 21 - \underline{\quad} - 10(\underline{\quad})$ $j^2 = -1$

$= \underline{\quad} - j$ Add.

The voltage is $\underline{\quad\quad}$ volts.

Example 7 Divide Complex Numbers

Simplify $\dfrac{5i}{3 + 2i}$.

Rationalize the denominator to simplify the fraction.

$\dfrac{5i}{3 + 2i} = \dfrac{5i}{3 + 2i} \cdot \dfrac{3 - 2i}{3 - 2i}$ $3 + 2i$ and $3 - 2i$ are complex conjugates.

$= \dfrac{15i - 10i^2}{9 - 4i^2}$ Multiply the numerator and denominator.

$= \dfrac{15i - 10(-1)}{9 - 4(-1)}$ $i^2 = -1$

$= \dfrac{15i + 10}{13}$ Simplify.

$= \dfrac{10}{13} + \dfrac{15}{13}i$ $a + bi$ form

Check

Simplify $\dfrac{2i}{-4 + 3i}$ $\underline{\quad\quad\quad}$