



Solving Quadratic Equations by Graphing

Explore Roots of Quadratic Equations

 **Online Activity** Use graphing technology to complete the Explore.

 **INQUIRY** How can you use the graph of a quadratic function to find the solutions of its related equation?

Today's Standards

A.CED.2, F.IF.4

MP4, MP5

Today's Vocabulary

quadratic equation

standard form of a quadratic equation

Learn Solving Quadratic Equations by Graphing

A **quadratic equation** is an equation that includes a quadratic expression.

Key Concept • Standard Form of a Quadratic Equation

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$ and a , b , and c are integers.

One method for finding the roots of a quadratic equation is to find the zeros of a related quadratic function. You can identify the solutions or roots of an equation by finding the x -intercepts of the graph of a related function. Often, exact roots cannot be found by graphing. You can estimate the solutions by finding the integers between where the zeros are located on the graph of the related function.

Example 1 One Real Solution

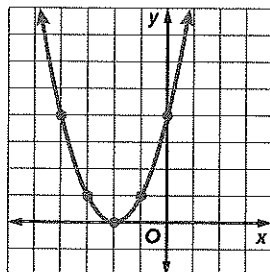
Solve $10 - x^2 = 4x + 14$ by graphing.

Solve the equation for 0. $0 = x^2 + \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$

Find the axis of symmetry. $x = -\frac{b}{2a} = -\frac{\hspace{1cm}}{2(\hspace{1cm})} = \underline{\hspace{1cm}}$

Make a table of values, plot the points, and connect them with a curve.

x	y
-4	_____
-3	_____
-2	_____
-1	_____
0	_____



The zero of the function is ____.

Therefore, the solution of the equation is ____ or $\{x \mid x = \underline{\hspace{1cm}}\}$.

Check

Solve $x^2 + 7x = 31x - 144$ by graphing. $x = \underline{\hspace{1cm}}$

Think About It!

How can you determine the number of solutions of a quadratic equation?

Study Tip:

Solutions of Quadratic Equations

A quadratic equation can have one real solution, two real solutions, or no real solutions.

Think About It!

How can you find the solution of the equation from the table?



Go Online You can watch a video to see how to solve quadratic equations by graphing on a graphing calculator.



Think About It! Explain why 9 and 15 cannot be solutions, even though their sum is 24.

Example 2 Two Real Solutions

NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of 24 and a product of 119.

UNDERSTAND What do you know?

Let x represent one of the numbers. Then $24 - x$ will represent the other number. So $x(24 - x) = 119$.

What do you need to find?

x and $24 - x$

PLAN AND SOLVE

Step 1 Solve the equation for 0.

$$x(24 - x) = 119 \quad \text{Original equation}$$

$$24x - x^2 = 119 \quad \text{Distributive Property}$$

$$0 = x^2 - 24x + 119 \quad \text{Subtract } 24x - x^2 \text{ from each side.}$$

Step 2 Find the axis of symmetry.

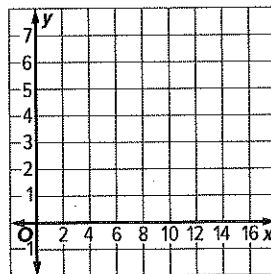
$$x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}$$

$$x = -\frac{-24}{2(1)} \quad a = 1, b = -24$$

$$x = 12 \quad \text{Simplify.}$$

Step 3 Make a table of values and graph the function.

x	y
14	
13	
12	
11	
10	



Steps 4 and 5 Find the zero(s) and determine the solution.

The zeros of the function are $x = 11$ and $x = 13$.

$x = 11$ or $x = 13$, so $24 - x = 13$ or $24 - x = 11$. Thus, the two numbers with a sum of 24 and a product of 119 are 11 and 13 .

Check

NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of -43 and a product of 306 . $x = -18$ and $x = -25$



Go Online You can complete an Extra Example online.

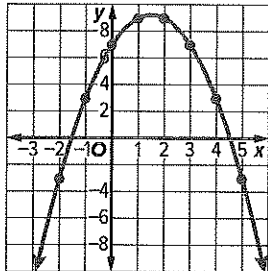
Example 3 Estimate Roots

Solve $-x^2 + 4x + 7 = 0$ by graphing. If the exact roots cannot be found, state the consecutive integers between which the roots are located.

Find the axis of symmetry. $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

Make a table of values, plot the points, and connect them with a curve.

x	y
-2	—
-1	—
0	—
1	—
2	—
3	—
4	—
5	—



The x-intercepts of the graph indicate that one solution is between ____ and ____, and the other solution is between ____ and ____.

Check

Use a graph to find all of the solutions of $x^2 + 9x - 5 = 0$. Select all of the pairs of consecutive integers between which the roots are located.

Example 4 Solve by Using a Table

Use a table to solve $-x^2 + 5x - 1 = 0$.

Steps 1 and 2 Enter the function and view the table.

Enter $-x^2 + 5x - 1$ in the **Y=** list. Use the **TABLE** window to find where the sign of **Y1** changes. The sign changes between $x = 0$ and $x = \underline{\hspace{1cm}}$.

X	Y1
-3	-25
-2	-15
-1	-7
0	-1
1	4
2	9
3	14
4	19
5	24
6	29
7	34
8	39
9	44
10	49
11	54
12	59
13	64
14	69
15	74
16	79
17	84
18	89
19	94
20	99
21	104
22	109
23	114
24	119
25	124
26	129
27	134
28	139
29	144
30	149
31	154
32	159
33	164
34	169
35	174
36	179
37	184
38	189
39	194
40	199
41	204
42	209
43	214
44	219
45	224
46	229
47	234
48	239
49	244
50	249
51	254
52	259
53	264
54	269
55	274
56	279
57	284
58	289
59	294
60	299
61	304
62	309
63	314
64	319
65	324
66	329
67	334
68	339
69	344
70	349
71	354
72	359
73	364
74	369
75	374
76	379
77	384
78	389
79	394
80	399
81	404
82	409
83	414
84	419
85	424
86	429
87	434
88	439
89	444
90	449
91	454
92	459
93	464
94	469
95	474
96	479
97	484
98	489
99	494
100	499

Steps 3 and 4 Edit the table settings and find a more accurate location.

Use **TBLSET** to change ΔTbl to 0.1 and look again for the sign change.

Repeat this for 0.1 and 0.001 to get a more accurate location of one zero.

One zero is located at approximately $x = \underline{\hspace{1cm}}$

X	Y ₁	
.206	-.0124	
.207	-.0078	
.208	-.0032	
.209	.0014	
.21	.0059	
.211	.01048	
.212	.01506	

Y₁ = .001319

Think About It!
How can you check your solutions?

Steps 5 and 6 Repeat the process and determine the solutions of the equation.

The zeros of the function are at approximately 0.209 and 4.791, so the solutions to the equation are approximately _____ and _____.

X	Y1
4.788	.01506
4.789	.01048
4.79	.0059
4.791	.001319
4.792	-.0033
4.793	-.0078
4.794	-.0124
Y1=.001319	

Check

Use a table to find all of the solutions of $-x^2 - 3x + 8 = 0$.

_____ and _____

Example 5 Solve by Using a Calculator

FOOTBALL A kicker punts a football. If the ball is 1.5 feet above the ground when his foot meets the ball, how long will it take the ball to hit the ground? Use the formula $h(t) = -16t^2 + 50t + h_0$, where t is the time in seconds and h_0 is the initial height.

We know that h_0 is the initial height, so $h_0 = 1.5$. We need to find t when $h(t)$ is 0. Use a graphing calculator to graph the related function $h(t) = -16t^2 + 50t + \underline{\hspace{1cm}}$

Step 1 Enter the function in the $Y=\text{list}$, and press graph.

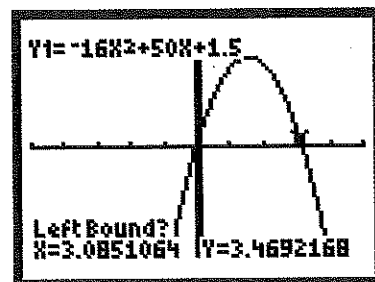
Step 2 Use the zero feature in the CALC menu to find the positive zero.

Step 3 Find the left bound.

Step 4 Find the right bound.

Step 5 Find and interpret the solution.

The zero is approximately _____. Thus, the ball hit the ground approximately _____ seconds after it was punted.



Check

SOCCER A goalie punts a soccer ball. If the ball is 1 foot above the ground when her foot meets the ball, find how long it will take, to the nearest hundredth of a second, for the ball to hit the ground. Use the formula $h(t) = -16t^2 + 35t + h_0$, where t is the time in seconds and h_0 is the initial height.

_____ seconds

Go Online You can complete an Extra Example online.