Solving Systems of Equations Algebraically

Learn Solving Systems of Equations in Two Variables by Substitution

Key Concept • Substitution Method

Step 1 When necessary, solve at least one equation for one of the variables.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation to solve for the other variable. Write the solution as an ordered pair.

Example 1 Substitution When There Is One Solution

Use substitution to solve the system of equations.

$8x - 3y = -1$	Equation 1
$x + 2y = -12$	Equation 2

Step 1 Solve one equation for one of the variables.

Because the coefficient of x in Equation 2 is 1, solve for x in that equation.

$x + 2y = -12$	Equation 2
$x = \_\_\_\_ y - \_\_\_\_$	Subtract $2y$ from each side.

Step 2 Substitute the expression.

Substitute for x in Equation 1. Then solve for y.

$8x - 3y = -1$	Equation 1
$8(\_\_\_\_\_\_) - 3y = -1$	$x = -2y - 12$
$\_\_\_\_ y - \_\_\_\_ - 3y = -1$	Distributive Property
$\_\_\_\_ y - 96 = -1$	Simplify.
$-19y = \_\_\_\_$	Add 96 to each side.
$y = \_\_\_\_$	Divide each side by -19.

Step 3 Substitute to solve.

Substitute the value of y into one of the original equations to solve for x.

$x + 2y = -12$	Equation 2
$x + 2(\_\_\_\_) = -12$	$y = -5$
$x - \_\_\_\_ = -12$	Multiply.
$x = \_\_\_\_$	

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Today's Standards
A.CED.3
MP1, MP6

Today's Vocabulary
substitution
elimination

Go Online You can learn how to solve a system of equations by using substitution with algebra tiles by watching the video online.

Talk About It!
Describe the benefit of solving a system of equations by substitution instead of graphing when the coefficients are not integers.

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Your Notes

Think About It!
What can you conclude about the slopes and y-intercepts of the equations when a system of equations has no solution? when a system of equations has infinitely many solutions?

Check

Use substitution to solve the system of equations.

$-5x + y = -3$

$3x - 8y = 24$ ______

Example 2 Substitution When There Is Not Exactly One Solution

Use substitution to solve the system of equations.

$\mathbf{-5x + 2.5y = -15}$	Equation 1
$\mathbf{y = 2x - 11}$	Equation 2

Equation 2 is already solved for y, so substitute $2x - 11$ for y in Equation 1.

$-5x + 2.5y = -15$	Equation 1
$-5x + 2.5(\_\_\_\_) = -15$	$y = 2x - 11$
$-5x + \_\_x - \_\_\_ = -15$	Distributive Property
$\_\_\_\_ = -15$	False

This system has ______ because $-27.5 = -15$ is not true.

Example 3 Apply the Substitution Method

CHEMISTRY **Ms. Washington will need 300 milliliters of a 5% HCl solution for her class to use during a lab. If she has a 3.5% HCl solution and a 7% HCl solution, how much of each solution should she use in order to make the solution needed?**

Step 1 Write two equations in two variables.

Let x be the amount of 3.5% solution and y be the amount of 7% solution.

$x + y = 300$	Equation 1
$0.035x + 0.07y = 0.05(300)$	Equation 2

Step 2 Solve one equation for one of the variables.

$x + y = 300$	Equation 1
$x = \_\_\_ + 300$	Subtract y from each side.

Step 3 Substitute the resulting expression and solve.

$0.035x + 0.07y = 15$	Equation 2
$0.035 \_\_\_\_\_ + 0.07y = 15$	$x = -y + 300$
$\_\_\_\_y + \_\_\_ + 0.07y = 15$	Distributive Property
$0.035y = 4.5$	Simplify.
$y \approx \_\_\_\_$	Divide each side by 0.035.

Think About It!
Explain what approximations were made while solving this problem and how they affect the solution.

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Step 4 Substitute to solve for the other variable.

$x + y = 300$	Equation 1
$x + 128.57 \approx 300$	$y \approx 128.57$
$x \approx 171.43$	Simplify.

The solution of the system is (171.43, 128.57). Ms. Washington should use 171.43 mL of the 3.5% solution and 128.57 mL of the 7% solution.

Learn Solving Systems of Equations in Two Variables by Elimination

Key Concept • Elimination Method

Step 1 Multiply one or both of the equations by a number to result in two equations that contain opposite or equal terms.

Step 2 Add or subtract the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example 4 Elimination When There Is One Solution

Use elimination to solve the system of equations.

$\mathbf{-2x - 9y = -25}$	Equation 1
$\mathbf{-4x - 9y = -23}$	Equation 2

Step 1 Multiply the equations.

Multiply Equation 2 by −1 to get opposite terms $-9y$ and $9y$.

$-4x - 9y = -23 \xrightarrow{\text{Multiply by } -1.} \underline{\quad}x + \underline{\quad}y = \underline{\quad}$

Step 2 Add the equations.

Add the equations to eliminate the y-term and solve for x.

$-2x - 93 = -25$	Equation 1
$(+)\ 4x + 9y = 23$	Equation 2 × (−1)
$\underline{\quad}x = \underline{\quad}$	Add the equations.
$x = \underline{\quad}$	Divide each side by 2.

Step 3 Substitute and solve.

$-4x - 9y = -23$	Substitute −1 for x in Equation 2.
$-4(\underline{\quad}) - 9y = -23$	$x = -1$
$4 - 9y = -23$	Multiply.
$\underline{\quad}y = \underline{\quad}$	Subtract 4 from each side.
$y = 3$	Divide each side by −9.

The solution of the system is (___, ___)

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Think About It!
When using elimination, when should you add the equations, and when should you subtract the equations?

Think About It!
Describe the benefit of using elimination instead of substitution of this problem.

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Example 5 Multiply Both Equations Before Using Elimination

Use elimination to solve the system of equations.

$2x + 5y = 1$	Equation 1
$3x - 4y = -10$	Equation 2

Step 1 Multiply one or both equations.

Multiply Equation 1 by 3 and Equation 2 by 2.

$2x + 5y = 1$	Equation 1	$3x - 4y = -10$	Equation 2
$3(2x + 5y) = 3(1)$	Mult. by 3.	$2(3x - 4y) = 2(-10)$	Mult. by 2.
$\underline{\quad}x + \underline{\quad}y = \underline{\quad}$	Simplify.	$\underline{\quad}x + \underline{\quad}y = \underline{\quad}$	Simplify.

Step 2 Eliminate one variable and solve.

In order to eliminate the x-terms, subtract the equations. Then, solve for y.

$6x + 15y = 3$	Equation 1 × 3
$(-)\ 6x - 8y = -20$	Equation 2 × 2
$\underline{\quad}y = \underline{\quad}$	Subtract the equations.
$y = \underline{\quad}$	Divide each side by 23.

Step 3 Substitute and solve.

Substitute $y = 1$ in either of the original equations and solve for x.

$2x + 5y = 1$	Equation 1
$2x + 5(1) = 1$	$y = 1$
$2x + 5 = 1$	Multiply.
$x = \underline{\quad}$	Solve for x.

The solution of the system is $(\underline{\quad}, \underline{\quad})$.

Think About It!
Describe the graph of this system of equations.

Example 6 Elimination Where There is Not Exactly One Solution

Use elimination to solve the system of equations.

$18x + 21y = 14$	Equation 1
$6x + 7y = 2$	Equation 2

Steps 1 and 2 Multiply one or both equations and add them.

Multiply Equation 2 by −3. Then add the equations.

$$18x + 21y = 14$$
$$6x + 7y = 2 \xrightarrow{\text{Multiply by } -3.} \begin{array}{r} 18x + 21y = 14 \\ (+)\ -18x - 21x = -6 \\ \hline 0 \neq 8 \end{array}$$

Because $0 \neq 8$, this system has no solution.

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Go Online to practice what you've learned in Lessons 2-4 and 2-5.

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