Equations of Linear Functions

Today's Standards
A.CED.2, F.IF.6
MP1, MP2, MP4

Today's Vocabulary
standard form of a linear equation
slope-intercept form
point-slope form

Explore Arithmetic Sequences

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you write formulas that relate to the numbers in an arithmetic sequence?

Learn Linear Equations in Standard Form

Any linear equation can be written in **standard form**, $Ax + By = C$, where $A \geq 0$, A and B are not both 0, and A, B, and C are integers with a greatest common factor of 1.

Think About It!
Is $-2x + 2y = 2$ written in standard form? Why or why not?

Example 1 Write Linear Equations in Standard Form

Write $y = \frac{2}{5}x + 14$ in standard form. Identify A, B, and C.

$y = \frac{2}{5}x + 14$	Original equation
$-\frac{2}{5}x + y = 14$	Subtract $\frac{2}{5}x$ from each side.
$\_\_\_x - \_\_\_y = \_\_\_$	Multiply each side by -5.

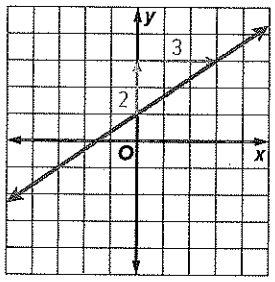
$A =$ ___ $B =$ ___ $C =$ ___

Learn Linear Equations in Slope-Intercept Form

Any linear equation can be written in **slope-intercept form,** $y = mx + b$ where m is the slope and b is the y-intercept.

The slope is $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$. This value can be substituted for m in the slope-intercept form.

The line intersects the y-axis at 1. This value can be substituted for b in the slope-intercept form.



Think About It!
Is the b in slope-intercept form equivalent to the B in standard form, $Ax + By = C$? If yes, explain your reasoning. If no, provide a counterexample.

Go Online You can complete an Extra Example online.

Your Notes

Example 2 Write Linear Equations in Slope-Intercept Form

Write $12x - 4y = 24$ in slope-intercept form. Identify the slope m and y-intercept b.

$12x - 4y = 24$	Original equation
$-4y = -12x + 24$	Subtract $12x$ from each side.
$y = \_\_\_x - \_\_\_$	Divide each side by -4.

$m = \_\_\_$ $b = \_\_\_$

Check

Write $4x = -2y + 22$ in slope-intercept form. $y = -\_\_\_x + \_\_\_$

Example 3 Interpret an Equation in Slope-Intercept Form

SHOES **The equation $3246x - 2y = -152{,}722$ can be used to estimate shoes sales in Europe from 2010 to 2015, where x is the number of years after 2010 and y is the revenue in millions of dollars.**

Part A Write the equation in slope-intercept form.

$3246x - 2y = -152{,}722$	Original equation
$-2y = -3246x - 152{,}722$	Subtract $3246x$ from each side.
$y = \_\_\_\_\_x + \_\_\_\_\_$	Divide each side by -2.

Part B Interpret the parameters in the context of the situation.

1623 represents that sales increased by $_____ each year.

76,361 represents that in year 0, or in _____, sales were $_____.

Example 4 Use a Linear Equation in Slope-Intercept Form

SMARTPHONES **In 2013, there were 1.31 billion smartphone users worldwide. By 2017, there were 2.38 billion smartphone users. Write and use an equation to estimate the number of users in 2025.**

Step 1 Define the variables. Because you want to estimate the number of users in 2025, write an equation that represents the number of smartphone users y after x years. Let x be the number of years after 2013 and let y be the number of billions of smartphone users.

Go Online You can complete an Extra Example online.

Think About It!

When using the equation to estimate the number of smartphone users in the future, what constraint does the world's population place on the possible number of users?

Study Tip

Assumptions

Assuming that the rate at which the number of smartphone users increases is constant allows us to represent the situation using a linear equation. While the rate at which the number of smartphone users increases may vary each year, using a constant rate allows for a reasonable equation that can be used to estimate future data.

Step 2 Find the slope. Since x is the years after 2013, (0, 1.31) and (4, 2.38) represent the number of smartphone users in 2013 and 2017, respectively. Round to the nearest hundredth.

$$m = \frac{2.38 - \_\_\_}{\_\_\_} = \_\_\_\_$$

So, the number of users is increasing at a rate of _____ billion per year.

Step 3 Find the *y*-intercept. The y-intercept represents the number of smartphone users when $x = 0$, or in 2013. So, $b =$ _____

Step 4 Write an equation. Use $m = 0.27$ and $b = 1.31$ to write the equation.

$y =$ _____$x +$ _____ $\qquad m = 0.27, b = 1.31$

Step 5 Estimate. Since 2025 is 12 years after 2013, substitute 12 for x.

$y = 0.27(12) + 1.31; y = 4.55$

If the trend continues, there will be about 4.55 billion users in 2025.

Think About It!

Suppose the data spanned 2 years instead of 4 years. That is, there were 1.31 billion smartphone users in 2013 and 2.38 billions users in 2015. How would this affect the rate of change abnd your estimate in **Step 5**?

Learn Linear Equations in Point-Slope Form

Any linear equation can be written in **point-slope form**, $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a point on the line.

Example 5 Point-Slope Form Given Slope and One Point

Write the equation of a line that passes through (3, −5) and has a slope of 11 in point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-5) = 11(x - 3)$	$m = 11; (x_1, y_1) = (3, -5)$
$y + 5 = 11(x - 3)$	Simplify.

Talk About It

What other values would you need to write the equation of this line in slope-intercept form? Could you determine those values given the information above?

Example 6 Point-Slope Form Given Two Points

Write an equation of a line that passes through (1, 1) and (7, 13) in point-slope form.

Step 1 Find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Point-slope form
$= \frac{13 - \_\_\_}{\_\_\_}$	$(x_1, y_1) = (1, 1); (x_2, y_2) = (7, 13)$
$= \frac{12}{\_\_\_}$	Simplify.
$=$ _____	Simplify.

(*continued on the next page*)

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Step 2 Write an equation.

Substitute the slope for m and the coordinates of either of the given points for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = 2(x - 1)$ $m = 2$; $(x_1, y_1) = (1, 1)$

Check ____, ____

Select all the equations with lines that pass through (–1, 1) and (–2, 13).

A. $x - 1 = -12(y + 1)$

B. $y - 1 = -12(x + 1)$

C. $x + 1 = -12(y - 1)$

D. $y + 1 = -12(x - 1)$

E. $y - 2 = -12(x + 13)$

F. $x - 2 = -12(y + 13)$

G. $x + 2 = -12(y - 13)$

H. $y + 2 = -12(x - 13)$

Example 7 Write and Interpret a Linear Equation in Point-Slope Form

ARCHITECTURE **The Tower of Pisa leaned 5.4 meters in 1993 compared to a lean of just 1.4 meters in 1350. Write an equation in point-slope form that represents the lean *y* of the Tower of Pisa *x* years after its construction in 1178.**

Step 1 Find the slope. Round to the nearest hundredth.

The tower was leaning 1.4 meters in 1350, _____ years after 1178.

The tower was leaning 5.4 meters in 1993, _____ years after 1178.

$m = \frac{5.4 -}{\quad} =$ _____ meter per year

Step 2 Write an equation.

Substitute the slope for m and the coordinates of either of the given points for (x_1, y_1) in the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y -$ ____ $=$ ____ $(x -$ ____$)$ $m = 0.006$; $(x_1, y_1) = (172, 1.4)$

Check

SOCIAL MEDIA In 2011, the Miami Marlins had about 11,000 followers on a social media site. In 2016, they had about 240,000 followers. Which equation represents the number of followers y the Miami Marlin's had x years after they joined the site in 2009? ____

A. $y - 11{,}000 = 45{,}800\ (x - 2)$

B. $y - 45{,}800 = 11{,}000\ (x - 2)$

C. $y - 11{,}000 = 45{,}800\ (x - 2011)$

D. $y - 2 = 45{,}800\ (x - 11{,}000)$

Go Online You can complete an Extra Example online.

Think About It!

Could this equation be used to estimate the lean of the Tower of Pisa for any year? Explain your reasoning.