


Solving Linear Equations and Inequalities

Explore Comparing Linear Equations and Inequalities

 **Online Activity** Use a comparison to complete the Explore.

 **INQUIRY** How do the solution methods and the solutions of linear equations and inequalities in one variable compare?

Today's Standards

A.CED.1, A.CED.2

MP1, MP2

Today's Vocabulary

equation

solution

root

zero

inequality

Learn Solving Linear Equations

An **equation** is a mathematical sentence stating that two mathematical expressions are equal. The **solution** of an equation is a value that makes the equation true. To solve equations, use the properties of equality to isolate the variable on one side.

| Property of Equality | Symbols |
|-------------------------------------|---|
| Addition Property of Equality | For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$. |
| Subtraction Property of Equality | For any real numbers a , b , and c , if $a = b$, then $a - c = b - c$. |
| Multiplication Property of Equality | For any real numbers a , b , and c , $c \neq 0$, if $a = b$, then $ac = bc$. |
| Division Property of Equality | For any real numbers a , b , and c , $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$. |

Example 1 Solve a Linear Equation

Solve $\frac{1}{3}(2x - 57) + \frac{1}{3}(6 - x) = -4$.

$$\frac{1}{3}(2x - 57) + \frac{1}{3}(6 - x) = -4 \quad \text{Original equation}$$

$$\frac{2}{3}x - \underline{\hspace{1cm}} + \underline{\hspace{1cm}} - \frac{1}{3}x = -4 \quad \text{Distributive Property}$$

$$\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = -4 \quad \text{Combine like terms.}$$

$$\frac{1}{3}x = \underline{\hspace{1cm}} \quad \text{Add 17 to each side and simplify.}$$

$$x = \underline{\hspace{1cm}} \quad \text{Multiply each side by 3 and simplify.}$$

Check

$$\text{Solve } -2(-4n + 1) + 5\left(\frac{2}{5}n - 8\right) = -62.$$

$$n = \underline{\hspace{1cm}}$$

 **Go Online** You can complete an Extra Example online.

Study Tip

Justifications The properties of equality are used as justifications above. However, in most future solutions, the justifications for steps will read as "Subtract c from each side," or "Divide each side by c ."

Talk About It!

In the last step, why is each side of the equation multiplied by 3? How is this related to the Division Property of Equality?

Go Online

You can learn how to use algebra tiles to solve equations in one variable by watching the video online.

Example 2 Write and Solve an Equation

SPACE The diameter of Earth is 828 kilometers less than twice the diameter of Mars. If Earth has a diameter of 12,756 kilometers, what is the diameter of Mars?

Part A Write an equation that represents the situation.

Words The diameter of Earth is 828 less than twice the diameter of Mars.

Variable Let $m =$ _____

Equation _____ $= 2m -$ _____

Part B Solve the equation.

$$12,756 = 2m - 288 \quad \text{Original equation}$$

$$12,756 + 288 = 2m - 288 + 288 \quad \text{Add 288 to each side.}$$

$$13,044 = 2m \quad \text{Simplify.}$$

$$\frac{13,044}{2} = \frac{2m}{2} \quad \text{Divide each side by 2.}$$

$$6,522 = m \quad \text{Simplify.}$$

The diameter of Mars is _____ kilometers. This is a reasonable solution because $12,756$ is a little less than $6,522 \cdot 2 = 13,044$, as indicated in the problem.

Check

BASKETBALL In 1962, Wilt Chamberlain set the record for the most points scored in a single NBA game. He scored 28 points from free throws and made x field goals, worth two points each. If Wilt Chamberlain scored 100 points, how many field goals did he make? Which equation represents the number of field goals that Chamberlain scored? _____

A. $100 = 28 + 2x$ B. $100 = 2x$ C. $28 = 2x$ D. $100 = 2x$

How many field goals did Chamberlain score? _____ field goals

Example 3 Solve for a Variable

GEOMETRY The formula for the perimeter of a parallelogram is $P = 2a + 2b$ where a and b represent the measures of the bases. Solve the equation for b .

$$P = 2a + 2b \quad \text{Original equation}$$

$$P - 2a = 2a + 2b - 2a \quad \text{Subtract } 2a \text{ from each side.}$$

$$P - 2a = 2b \quad \text{Simplify.}$$

$$\frac{P - 2a}{2} = \frac{2b}{2} \quad \text{Divide each side by 2.}$$

$$\frac{P - 2a}{2} = b \quad \text{Simplify.}$$

Go Online You can complete an Extra Example online.

Think About It!
What does it mean to solve for a variable?

Check

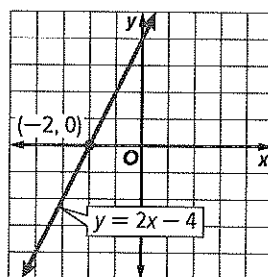
GEOMETRY The formula for the area A of a trapezoid is solved for h . Fill in the missing justification.

$$\begin{aligned} A &= \frac{1}{2}h(a + b) && \text{Original equation.} \\ 2A &= 2 \cdot \frac{1}{2}h(a + b) && \text{Multiplication Property of Equality} \\ 2A &= h(a + b) && \text{Simplify.} \\ \frac{2A}{(a + b)} &= \frac{h(a + b)}{(a + b)} && \text{Divide both sides by } (a + b). \\ \frac{2A}{(a + b)} &= h && \text{Simplify.} \end{aligned}$$

Learn Solving Linear Equations by Graphing

| Equation | Related Function |
|---------------|---------------------------------|
| $2x + 13 = 9$ | $f(x) = 2x + 4$ or $y = 2x + 4$ |

The solution of an equation is called a **root**. You can find the root of an equation by examining the graph of its related function $f(x)$. Values of x for which $f(x) = 0$ are called **zeros** of the function f . The zero of a function is the x -intercept of its graph. The solution and root of a linear equation are the same as the zero and x -intercept of its related function.



Example 4 Solve a Linear Equation by Graphing

Step 1 Find a related function.

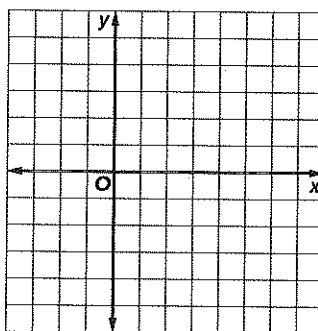
Rewrite the equation with 0 on the right side.

$$\begin{aligned} \frac{1}{2}x - 11 &= -8 && \text{Original equation.} \\ \frac{1}{2}x - 11 + 8 &= -8 + 8 && \text{Add 8 to each side.} \\ \frac{1}{2}x - 3 &= 0 && \text{Simplify.} \end{aligned}$$

Replacing 0 with $f(x)$ gives the related function $f(x) = \frac{1}{2}x - 3$.

Step 2 Graph the related function.

Since the graph of $f(x) = \frac{1}{2}x - 3$ intersects the x -axis at $\underline{\hspace{1cm}}$, the solution of the equation is $\underline{\hspace{1cm}}$.



Talk About It!

Because there is typically more than one way to solve an equation for 0, there may be more than one related function for an equation. What is another possible related function of $2x + 13 = 9$? How does the zero of this function compare to the zero of $f(x) = 2x + 4$?

Think About It!

Explain why -3 is *not* a zero of the function.

Watch Out!

Intercepts Be careful not to mistake y -intercepts for zeros of functions. The y -intercept on a graph occurs when $x = 0$. The x -intercepts are the zeros of a function because they are where $f(x) = 0$.

Use a Source

Use available resources to find Paul Crake's actual time. How does this compare to your solution?

Study Tip

Assumptions

Assuming that the rate at which Paul Crake climbed the stairs was constant allows us to represent the situation with a linear equation. While the rate at which he climbed likely varied throughout the race, using constant rates allows for reasonable graphs and solutions.

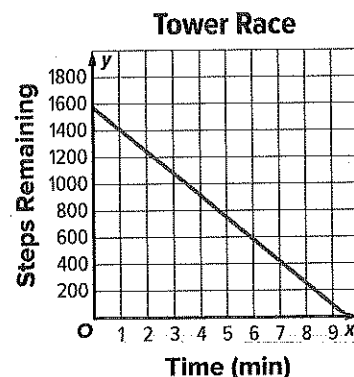
Example 5 Estimate Solutions by Graphing

TOWER RACE The Empire State Building Run-Up is a race in which athletes run up the building's 1576 stairs. In 2003, Paul Crake set the record for the fastest time, running up an average of about 165 stairs per minute. The function $c = 1576 - 165m$ represents the number of steps Crake had left to climb c after m minutes. Find the zero of the function and interpret its meaning in the context of the situation.

Step 1 Graph the function.

Step 2 Estimate the zero.

The graph appears to intersect the x -axis at about _____. This means that Paul Crake finished the race in about 9.5 minutes, or _____ minutes and _____ seconds.



Step 3 Solve algebraically.

Write the function as an equation set equal to 0, and solve algebraically to check your solution.

$$c = 1576 - 165m$$

Original equation

$$0 = 1576 - 165m$$

Replace c with 0.

$$\text{_____} = \text{_____}$$

Add $165m$ to each side.

$$m \approx \text{_____}$$

Divide each side by 165.

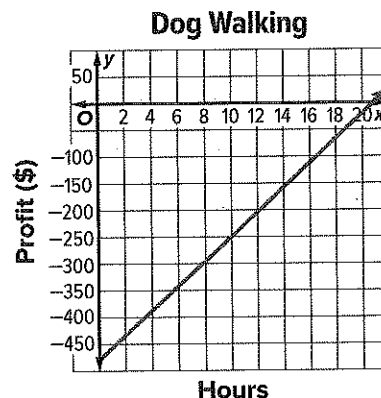
The solution is about _____. So, Paul Crake completed the Empire State Building Run-Up in about 9.55 minutes, or ____ minutes and ____ seconds. This is close to the estimated time of 9.5 minutes.

Check

DOG WALKING Bethany spends \$480 on supplies to start a dog walking service for which she plans to charge \$23 per hour. The function $y = 23x - 480$ represents Bethany's profit after x hours of dog walking.

Part A The graph appears to intersect the x -axis at about _____.

Part B Solve algebraically to verify your estimate. Round to the nearest hundredth. _____



Go Online You can complete an Extra Example online.

Learn Solving Linear Inequalities

An **inequality** is an open sentence that contains the symbol $<$, \leq , $>$, \geq . Properties of inequalities allow you to perform operations on each side of an inequality without changing the truth of the inequality.

Key Concept • Addition and Subtraction Properties of Inequality

Symbols For any real numbers a , b , and c :

If $a > b$, then $a + c > b + c$. If $a > b$, then $a - c > b - c$.

If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.

| | | |
|----------|-----------------|-----------------|
| Examples | $2 > 0$ | $9 > 6$ |
| | $2 + 1 > 0 + 1$ | $9 - 4 > 6 - 4$ |
| | $3 > 1$ | $5 > 2$ |

Key Concept • Multiplication and Division Properties of Inequality

Symbols For any real numbers a , b , and c :

where c is positive:

If $a > b$, then $ac > bc$. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$, then $ac < bc$. If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

where c is negative:

If $a > b$, then $ac < bc$. If $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$, then $ac > bc$. If $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

| | | |
|----------|-------------------------------|---------------------------------|
| Examples | $8 > -2$ | $-4 < -2$ |
| | $8(3) > -2(3)$ | $-4(-8) < -2(-8)$ |
| | $24 > -6$ | $32 > 16$ |
| | $-3 > -7$ | $12 > -4$ |
| | $\frac{-3}{2} > \frac{-7}{2}$ | $\frac{12}{-2} > \frac{-4}{-2}$ |
| | $-1.5 > -3.5$ | $-6 < 2$ |

The solution sets of inequalities can be expressed by using set-builder notation. For example, $\{x \mid x > 1\}$ represents the set of all numbers x such that x is greater than 1. The solution sets can also be graphed on number lines. Circles and dots are used to indicate whether an endpoint is included in the solution. The circle at 1 means that this point is *not* included in the solution set.



Go Online

You may want to complete the Concept Check to check your understanding.

Think About It!

What does the arrow on the graph of a solution set represent?

Think About It!

What does the dot on the graph of a solution set indicate?

Study Tip

Reversing the Inequality Symbol

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality. Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a negative number requires that the order of the inequality be reversed. In this problem, \geq was replaced with \leq .

Watch Out!

Reading Math Be sure to always read problems carefully. The term *at least* is used here, and can be confusing since it actually means *greater than or equal to*, and is represented by \geq . In this instance, Jake should intake at least 1300 mg, which means he must intake an amount greater than or equal to 1300 mg.

Example 6 Solve a Linear Inequality

Solve $-5.6n + 12.9 \geq -71.1$. Graph the solution set on a number line.

$$-5.6n + 12.9 \geq -71.1 \quad \text{Original inequality.}$$

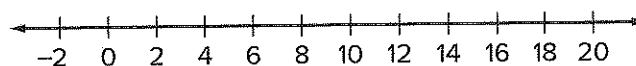
$$-5.6n + 12.9 \quad \geq \quad -71.1 \quad \text{Subtract 12.9 from each side.}$$

$$-5.6n \geq -84.4 \quad \text{Simplify.}$$

$$\underline{-5.6n} \leq \underline{-84} \quad \text{Divide each side by } -5.6, \text{ reversing the inequality symbol.}$$

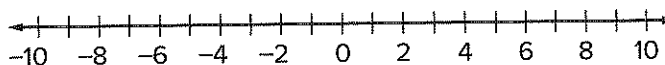
$$n \leq \quad \text{Simplify.}$$

The solution set is $\{n \mid n \leq \quad\}$. Graph the solution set.



Check

What is the solution of $-p - 3 \geq -4(p + 6)$? _____ Graph the solution set.



Example 7 Write and Solve an Inequality

NUTRITION The recommended daily intake of calcium for teens is 1300 mg. Jake gets 237 mg of calcium from a multivitamin he takes each morning and 302 mg from each glass of skim milk that he drinks. How many glasses of milk would Jake need to drink to meet the recommendation?

Step 1 Write an inequality to represent the situation.

Let g = the number of _____ Jake needs.

$$\quad + 302 \quad 1300$$

Step 2 Solve the inequality.

$$237 + 302g \geq 1300 \quad \text{Original inequality.}$$

$$302g \geq \quad \text{Subtract 237 from each side.}$$

$$g \geq \quad \text{Simplify.}$$

Step 3 Interpret the solution in the context of the situation.

Jake will need to drink slightly _____ glasses of milk to intake at least the recommended daily amount of calcium. This is a viable solution because Jake can pour part of a full glass of milk.

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