Graphing Linear Functions and Inequalities

Learn Graphing Linear Functions

The graph of a linear function represents all ordered pairs that are true for the function. You can use various methods to graph a linear function.

Example 1 Graph by Using a Table

Graph $x + 3y - 6 = 0$ by using a table.

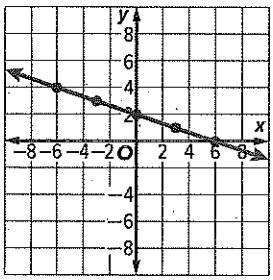
Solve the equation for y.

$x + 3y - 6 = 0$	Original function
$3y - 6 =$ _____	Subtract x from each side.
$3y = -x$ _____	Add 6 to each side.
$y =$ _____	Divide each side by 3.

Substitute each x-value into the equation to find the corresponding y-value.

Graph the ordered pairs in the table and draw a line through the points.

x	$-\frac{1}{3}x + 2$	y
-6	$-\frac{1}{3}(-6) + 2$	4
-3	$-\frac{1}{3}$ _____ $+ 2$	3
0	$-\frac{1}{3}(0) + 2$	_____
3	$-\frac{1}{3}(3) + 2$	_____
6	$-\frac{1}{3}$ _____ $+ 2$	0



Example 2 Graph by Using Intercepts

Graph $3x - 2y = -12$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

Case 1		Case 2
$3x - 2y = -12$	Original function	$3x - 2y = -12$
$3x - 2($_____$) = -12$	Replace with 0.	$3($_____$) - 2y = -12$
_____ $= -12$	Simplify.	_____ $= -12$
$x =$ _____	Divide.	$y =$ _____

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Today's Standards
A.CED.3; F.IF.4
MP5, MP6

Today's Vocabulary
linear inequality
boundary
closed half-plane
open half-plane
constraint

Watch the video online.

Study Tip
Recall that **slope** is the ratio of the change in the y-coordinates (rise) to the corresponding change in the x-coordinates (run) as you move from one point to another along a line.

Think About It!
Explain why -6, -3, 0, 3, and 6 were selected for the x-values in the table.

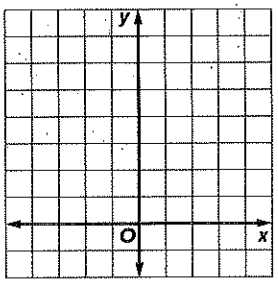
Your Notes

Think About It!
How can you check that the graph is correct?

The x-intercept is -4, and the y-intercept is 6. This means that the graph passes through (____, ____) and (____, ____).

Plot the two intercepts.

Draw a line through the points.



Example 3 Graph by Using the Slope and y-intercept

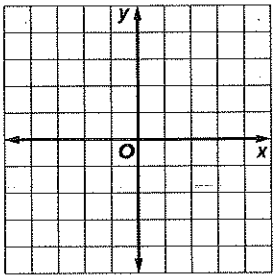
Graph $y = \frac{3}{2}x - 4$ by using m and b.

Follow these steps

- Begin by identifying the slope m and y-intercept b of the function.

  $m =$ ____ $b =$ ____

- Use the value of b to plot the y-intercept (____, ____).
- Use the slope of the line $m = \frac{3}{2}$ to plot more points. From the y-intercept, move up ____ units and ____ 2 units. Plot a point at (____, ____).
- From the point (2, -1), move ____ 3 units and right ____ units. Plot a point at (____, ____).
- Draw a line through the points.



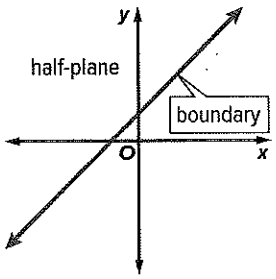
Explore Shading Graphs of Linear Inequalities

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you use a point to test the graph of an inequality?

Learn Graphing Linear Inequalities in Two Variables

The graph of a **linear inequality** is a half-plane with a boundary that is a straight line. The half-plane is shaded to indicate that all points contained in the region are solutions of the inequality. A **boundary** is a line or curve that separates the coordinate plane into two half-planes. The boundary is solid when the inequality contains $\leq$ or $\geq$ to indicate that the



Watch the videos online to learn how to graph functions using a graphing calculator.

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points on the boundary are included in the solution, creating a **closed half-plane**. The boundary is dashed when the inequality contains $<$ or $>$ because the points on the boundary do not satisfy the inequality. This results in an **open half-plane**.

A **constraint** is a condition that a solution must satisfy. Each solution of the inequality represents a viable, or possible, option that satisfies the constraint.

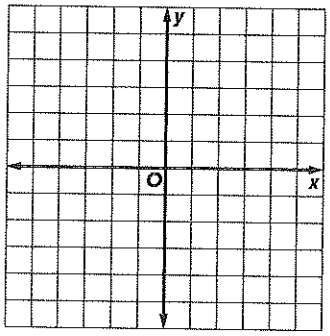
Example 4 Graph an Inequality with an Open Half-Plane

Graph $x + 4y < 12$.

Step 1 Graph the boundary.

$x + 4y < 12$	Original inequality
$x <$ ____ $+ 12$	Subtract $4y$ from each side.
x ____ $< -4y$	Subtract 12 from each side.
____ $x +$ ____ $> y$	Divide each side by -4, and reverse the inequality symbol.

The boundary of the graph is y ____ $-\frac{1}{4}x + 3$. Because the inequality symbol is $<$, the boundary is _____.



Step 2 Use a test point and shade.

Test (0, 0).

$x + 4y < 12$	Original inequality
____ $+ 4($____$) \overset{?}{<} 12$	Substitute values of test point (0, 0).
$0 < 12$	True.

Because (0, 0) is a solution of the inequality, shade the half-plane that contains the test point.

Check

You can check by selecting another point in the shaded region to test.

Example 5 Graph an Inequality with a Closed Half-Plane

Graph $9 + 3y \leq 8x$.

Step 1 Graph the boundary.

Solve for y in terms of x and graph the related function.

$9 + 3y \leq 8x$	Original inequality
$3y \leq 8x$ ____	Subtract 9 from each side.
$y \leq$ ____ $x -$ ____	Divide each side by 3.

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Watch the video online to see how to graph a linear inequality in two variables.

Study Tip

Above or Below

Usually the shaded half-plane of a linear inequality is said to be *above* or *below* the line of the related equation. However, if the equation of the boundary is $x = c$ for some constant c, then the function is a vertical line. In this case, the shading is considered to be *to the left* or *to the right* of the boundary.

Talk About It!

Can a linear inequality ever be a function? Explain your reasoning.

Think About It!

Why should you not test a point that is on the boundary?

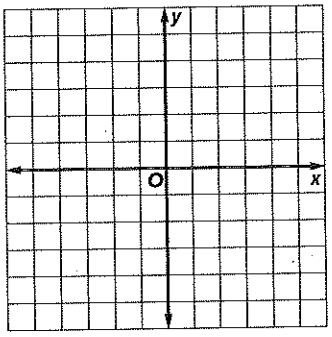
The related equation of $y \leq \frac{8}{3}x - 3$ is $y = \frac{8}{3}x - 3$, and the boundary is solid.

Step 2 Use a test point and shade.

Select a test point, such as (0, 0).

$9 + 3y \leq 8x$	Original inequality
$\_\_ + 3(\_\_) \stackrel{?}{\leq} 8(\_\_)$	$(x, y) = (0, 0)$.
$9 \not\leq 2$	False.

Shade the side of the graph that does not contain the test point.



Think About It!

Is (3, 5) a solution of the inequality? Explain.

Watch the video online to learn how to graph an inequality using agraphing calculator.

Example 6 Linear Inequalities

GRADES **Malik's algebra teacher determines semester grades by finding the sum of 70% of a student's test grade average and 30% of a student's homework grade average. If Malik wants a semester grade of 90% or better, write and graph the inequality that represents the constraints for Malik's test grade *x* and homework grade *y*.**

Understand

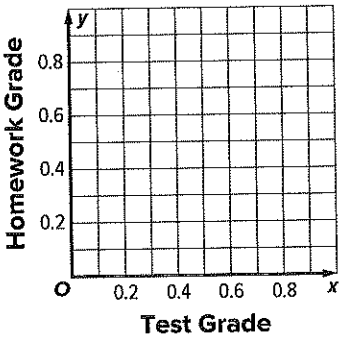
What do you know? What do you need to find?

Plan and Solve

Step 1 Write an inequality that represents the situation. _____

Step 2 Rearrange the inequality to solve for *y*. $y \geq$ _____

Step 3 Graph the related linear function as the boundary.



Step 4 Shade the half-plane. Use (0, 0) as a test point.

Step 5 Determine viable solutions for the inequality. Viable solutions that will result in Malik receiving an overall grade of at least 90% lie in the shaded region.

Check

How do you know that the shaded region is correct?

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