Sketching Graphs and Comparing Functions

Today's Standards
F.IF.4; F.IF.9
MP1, MP6

Explore Using Technology to Examine Key Features of Graphs

Online Activity Use graphing technology to complete the Explore.

INQUIRY What can key features of a function tell you about its graph?

Learn Sketching Graphs of Functions

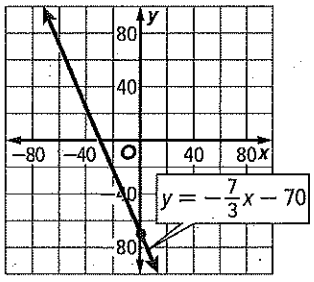
You can use key features of a function to sketch its graph.

Key Concept • Using Key Features

Key Feature	What it tells you about the graph
Domain	which values of x are defined for the function
Range	which values of y are defined for the function
Intercepts	where the graph crosses the x- or y-axes
Symmetry	where one side of the graph is a reflection or rotation of the other side
End Behavior	what the graph is doing at the right and left sides as x approaches infinity or negative infinity
Extrema	high or low points where the graph changes from increasing to decreasing or vice versa
Increasing/ Decreasing	where the graph is going up or down as x increases
Positive/Negative	where the graph is above or below the x-axis

Example 1 Sketch a Linear Function

Use the key features of the function to sketch its graph.



y-intercept: (0, −70)

Linearity: linear

Positive: for values of x such that $x < -30$

Decreasing: for all values of x

End Behavior: As $x \to \infty$, $f(x) \to -\infty$.
As $x \to -\infty$, $f(x) \to \infty$.

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Study Tip

Scales and Axes
Before you sketch a function, consider the scales or axes that best fit the situation. You want to capture as much information as possible, so you want the scales to be big enough to easily see the extrema and x- and y intercepts, but not so big that you cannot determine the values.

Talk About It!

Given the y-intercept and for what values of x the function is positive, what other information do you need to sketch a linear function? Explain your reasoning.

Your Notes

Example 2 Sketch a Nonlinear Function

Use the key features of the function to sketch its graph.

y-intercept: (0, 3)

Linearity: nonlinear

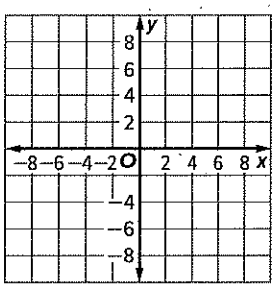
Continuity: continuous

Positive: for values of x

Decreasing: for all values of x such that $x < 0$

Extrema: relative minimum at (0, 3)

End Behavior: As $x \to \infty$, $f(x) \to \infty$.
As $x \to -\infty$, $f(x) \to \infty$.



Study Tip

Assumptions When sketching the function using the given key features, assumptions must be made. As in this example, the same key features could describe many different graphs. The key features could also be represented by a parabola, a curve that is narrower or wider, or an absolute value function.

Think About It!

Explain why the end behavior is not defined in the context of this situation.

Example 3 Sketch a Real-World Function

TEST DRIVE **Hae is test driving a car she is thinking of buying. She decides to accelerate to 60 miles per hour and then decelerate to a stop to test its acceleration and brakes. It takes her 15 seconds to reach her maximum speed and 15 additional seconds to come to a stop. Use the key features to sketch a graph that shows the speed *y* as a function of time *x*.**

***y*-intercept:** Hae starts her test drive at a speed of 0 miles per hour.

Linear or Nonlinear: The function that models the situation is nonlinear.

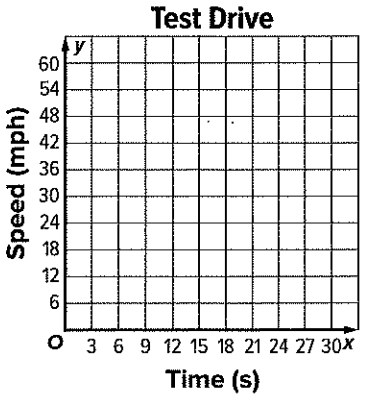
Extrema: Hae's maximum speed is 60 miles per hour, which she reaches 15 seconds into her test drive.

Increasing: Hae _____ the speed at a uniform rate for the first 15 seconds.

Decreasing: Hae decreases the speed at a _____ rate for the next 15 seconds until she reaches a _____.

End Behavior: Because Hae starts at ____ miles per hour and ends at ____ miles per hour, there is ____ end behavior.

Before sketching, consider the constraints of the situation. Hae cannot drive a negative speed or for a negative amount of time. Therefore, the graph only exists for positive x- and y-values.



Think About It!

Based on the graph, the speed of the car at 10 seconds is 40 miles per hour. Is it appropriate to assume that the car is traveling that exact speed at a specific time? Explain.

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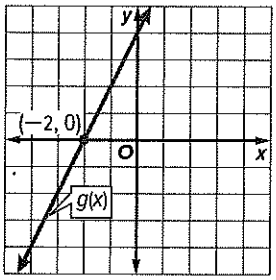
Example 4 Compare Properties of Linear Functions

Functions can be represented with a graph, a table, or by a verbal description. You can compare the properties and key features of functions represented in these different ways.

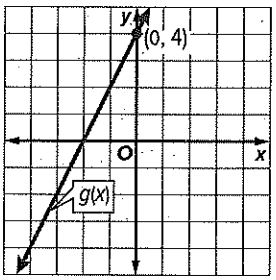
x	$f(x)$
−6	−3
−3	−2
0	−1
3	0
6	1



The x-intercept of $f(x)$ is ____, and the x-intercept of $g(x)$ is ____. The x-intercept of $f(x)$ is _____ than the x-intercept of $g(x)$.

So, $f(x)$ intersects the x-axis at a point farther to the _____ than $g(x)$.

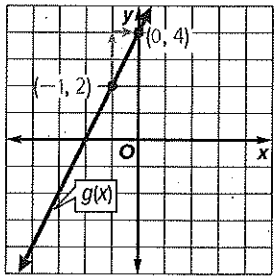
x	$f(x)$
−6	−3
−3	−2
0	−1
3	0
6	1



The y-intercept of $f(x)$ is ____, and the y-intercept of $g(x)$ is ____. The y-intercept of $g(x)$ is _____ than the y-intercept of $f(x)$.

So, $g(x)$ intersects the y-axis at a _____ point than $f(x)$.

	x	$f(x)$	
3	−6	−3	1
3	−3	−2	1
3	0	−1	1
3	3	0	1
3	6	1	1



The slope of $f(x)$ is $\frac{1}{3}$ and the slope of $g(x)$ is 2. Each function is increasing, but the slope of $g(x)$ is greater than the slope of $f(x)$.

So, $g(x)$ increases faster than $f(x)$.

Think About It!

How would a function that passes through (1, 0) with a slope of −4 compare to $f(x)$ and $g(x)$?

Pause and Reflect

Did you struggle with anything in this lesson? If so, how did you deal with it?

Record your observations here

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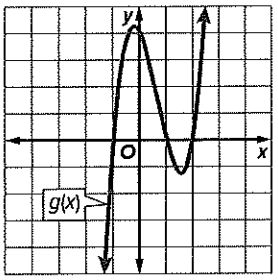
Example 5 Compare Properties of Nonlinear Functions

Examine the categories to see how to use the description and the graph to identify key features of each function. Then complete the statements to compare the two functions.

$f(x)$	$g(x)$
x-intercept: (−3.4, 0) y-intercept: (0, 1.5) relative maximum: (−2.3, 4.7) relative minimum: (−0.4, 1.1) end behavior: as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$	
x-intercepts	
$f(x)$ intersects the x-axis once at (−3.4, 0).	$g(x)$ intersects the x-axis three times at (−1, 0), (1, 0), and (2, 0).
y-intercept	
$f(x)$ intersects the y-axis at (0, 1.5).	$g(x)$ intersects the y-axis at (0, 4).
Extrema	
$f(x)$ has a relative maximum of 4.7 and a relative minimum of 1.1.	$g(x)$ has a relative maximum of about 4.2 and a relative minimum of about −1.2.
End Behavior	
As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$,	As $x \to -\infty$, $g(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$,

- The x-intercept of $f(x)$ is _____ any of the x-intercepts of $g(x)$.
- The graph of $g(x)$ intersects the x-axis _____ times than $f(x)$.
- The y-intercept of $f(x)$ is _____ the y-intercept of $g(x)$.
- So, $g(x)$ intersects the y-axis at a _____ point than $f(x)$.
- The relative maximum of $f(x)$ is _____ the relative maximum of $g(x)$. The relative minimum of $f(x)$ is _____ the relative minimum of $g(x)$.
- The two functions have _____ end behavior.

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