Extrema and End Behavior

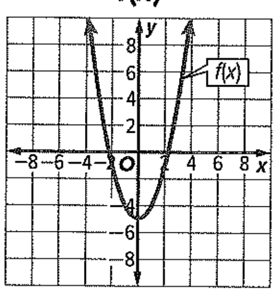
Learn Extrema of Functions

Graph of functions can have high and low points where they reach a maximum or minimum value. The maximum and minimum values of a function are called **extrema.** The **maximum** is at the highest point on the graph of a function. The **minimum** is at the lowest point on the graph of a function. The **relative maximum** is located at a point on the graph of a function where no other nearby points have a greater *y*-coordinate. The **relative minimum** is located at a point on the graph of a function where no other nearby points have a lesser *y*-coordinate.

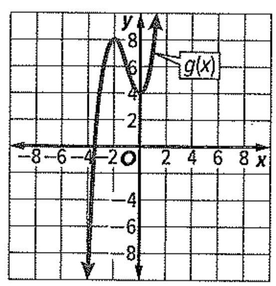
Example 1 Find Extrema from Graphs

Identify and estimate the *x*- and *y*-values of the extrema. Round to the nearest tenth if necessary.

$f(x)$



$g(x)$



$f(x)$: The function $f(x)$ is ________ as it approaches $x = 0$ from the left and ________ as it moves away from $x = 0$. Further, $(0, -5)$ is the lowest point on the graph, so $(0, -5)$ is a ________ .

$g(x)$: The function $g(x)$ is ________ as it approaches $x = -2$ from the left and ________ as it moves away from $x = -2$. Further, there are no greater y-coordinates surrounding $(-2, 8)$. However, $(-2, 8)$ is ____ the highest point on the graph, so $(-2, 8)$ is a ______ relative maximum.

The function $g(x)$ is ________ as it approaches $x = 0$ from the left and ________ as it moves away from $x = 0$. Further, there are no ______ y-coordinates surrounding $(0, 4)$. However, $(0, 4)$ is not the ______ point on the graph, so $(0, 4)$ is a ______ minimum.

Go Online You can complete an Extra Example online.

Today's Standards
F.IF.4; F.IF.7c
MP2, MP5

Today's Vocabulary
extrema
maximum
minimum
relative maximum
relative minimum
end behavior

Watch Out!

No Extrema Some functions, like $f(x) = x^3$, have no extrema.

Study Tip

Reading in Math In this context, *extrema* is the plural form of *extreme point*. The plural of *maximum* and *minimum* are *maxima* and *minima*, respectively.

Think About It!

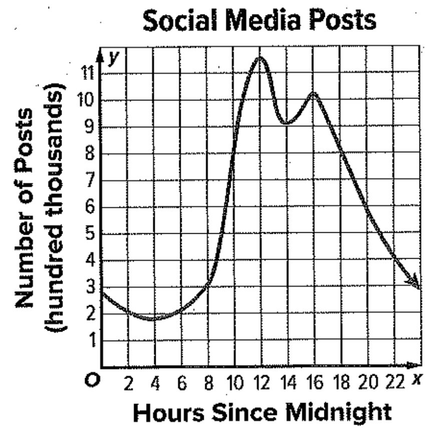
Why are the extrema identified on the graph of g(x) relative maxima and minima instead of maxima and minima?

Your Notes

Example 2 Find and Interpret Extrema

SOCIAL MEDIA Use the table and graph to estimate the extrema of the function that models the number of posts on a social media site in hundred thousands x given the number hours since midnight y. Describe the meaning of the extrema in the context of the situation.

x	y
0	2.8
4	1.8
8	3.1
12	11.5
14	9.1
16	10.2
20	5.8
24	2.8



maxima

The number of posts sent ____ hours after midnight is ________ than the number of posts made at any other time during the day. The highest point at the graph occurs when $x =$ ____ . Therefore, the maximum number of posts sent is about __________ at ______ noon.

minima

The number of posts sent ___ hours after midnight is _____ than the number of posts made at any other time during the day. The lowest point at the graph occurs when $x =$ ___ . Therefore, the minimum number of posts sent is about ________ at _________ .

relative maxima

The number of posts sent ____ hours after midnight is ________ than the number of posts during surrounding times, but is not the greatest number sent during the day. The graph has a relative peak when $x =$ ____ . Therefore, there is a relative peak in number of posts sent, or relative maximum, at _________ of about __________ posts.

relative maxima

The number of posts sent ____ hours after midnight is _____ than the number of posts during surrounding times, but is not the least number sent during the day. The graph dips when $x =$ ____ . Therefore, there is a relative low in number of posts sent, or relative minimum, at _________ of about ________ posts.

Explore End Behavior of Linear and Quadratic Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY Given the behavior of a linear or quadratic function as x increases towards infinity, how can you find the behavior as x decreases toward negative infinity or vice versa?

Go Online You can complete an Extra Example online.

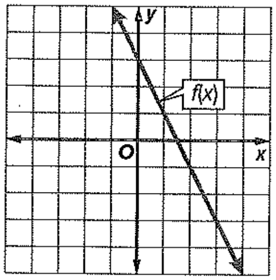
Learn End Behavior of Graphs of Functions

End behavior is the behavior of a graph as x approaches positive or negative infinity. As you move right along the graph, the values of x are increasing toward infinity. This is denoted as $x \to \infty$. At the left end, the values of x are decreasing toward negative infinity, denoted as $x \to -\infty$.

Example 3 End Behavior of Linear Functions

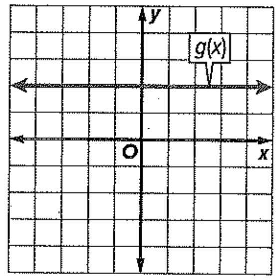
Use the graphs to describe the end behavior of each linear function.

a. ***f(x)***



As x decreases, $f(x)$ ________, and as x increases $f(x)$ ________.
Thus, as $x \to -\infty$, $f(x) \to$ ___ and as $x \to \infty$, $f(x) \to$ ____.

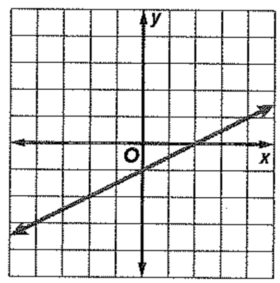
b. ***g(x)***



As x decreases or increases, $g(x) = 2$. Thus, as $x \to -\infty$, $g(x) =$ __, and as $x \to \infty$, $g(x) =$ __.

Check

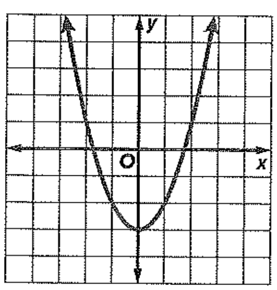
Use the graph to describe the end behavior of the function.



Example 4 End Behavior of Nonlinear Functions

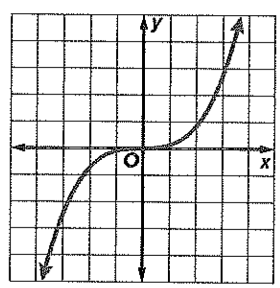
Use the graphs to describe the end behavior of each nonlinear function.

a. ***f(x)***



As you move left or right on the graph, $f(x)$ ________. Thus as $x \to -\infty$, $f(x) \to$ ___, and as $x \to \infty$, $f(x) \to$ ___.

b. ***g(x)***



As $x \to -\infty$, $g(x) \to$ _____, and as $x \to \infty$, $g(x) \to$ ___.

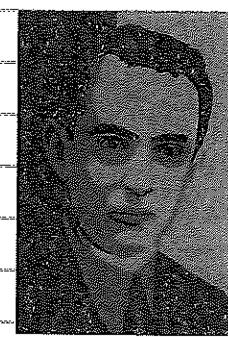
Go Online You can complete an Extra Example online.

Think About It!

For $f(x) = a$, where a is a real number, describe the end behavior of $f(x)$ as $x \to \infty$ and as $x \to -\infty$.

Talk About It!

In part **a**, the function's end behavior as $x \to -\infty$ is the opposite of the end behavior as $x \to \infty$. Do you think this is true for all linear functions where $m \neq 0$? Explain your reasoning.



Math History Minute

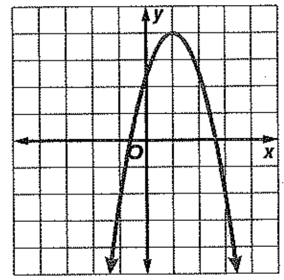
Júlio César de Mello e Souza (1895–1974) was a Brazilian mathematician who is known for his books on recreational mathematics. His most famous book, The Man Who Counted, includes problems, puzzles, and curiosities about math. The State Legislature of Rio de Janeiro declared that his birthday, May 6, be Mathematician's Day.

Think About It!

If the graph of a function is symmetric about a vertical line, what do you think is true about the end behavior of $f(x)$ as $x \to -\infty$ and as $x \to \infty$?

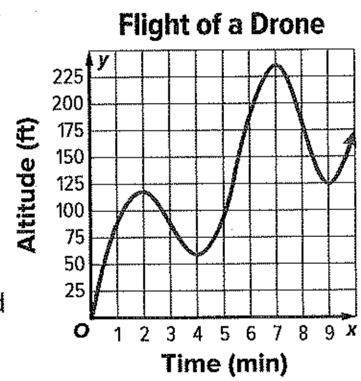
Check

Use the graphs to describe the end behavior of each function.



Example 5 Determine and Interpret End Behavior

DRONES **The graph shows the altitude of a drone above the ground $f(x)$ after x minutes. Describe the end behavior of $f(x)$ and interpret it in the context of the situation.**



Since the drone cannot travel for a negative amount of time, the function is not defined for $x <$ ___. So, there is ___ end behavior as $x \to$ ___.

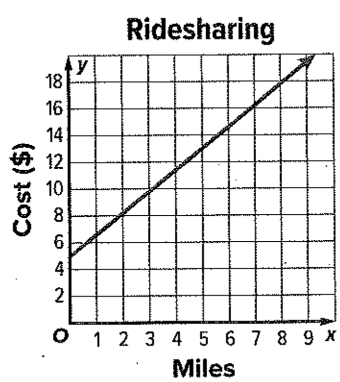
As $x \to \infty$, $f(x) \to$ ___. The longer the drone flies, the ___ it goes.

Study Tip:

Assumptions Assuming that the drone can continue to fly for an infinite amount of time and to an infinite altitude lets us analyze the end behavior as $x \to \infty$. While there are maximum legal altitudes that a drone can fly as well as limited battery life, assuming that the time and altitude will continue to increase allows us to describe the end behavior.

Check

RIDESHARING Mika and her friends are using a ride-sharing service to take them to a concert. The function models the cost of the ride $f(x)$ after x miles. Describe the end behavior of $f(x)$ and interpret it in the context of the situation.



Part A

What is the end behavior of the function? ___

A. as $x \to -\infty$, $f(x) \to -\infty$; as $x \to \infty$, $f(x) \to -\infty$

B. as $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to \infty$

C. as $x \to \infty$, $f(x) \to -\infty$; $f(x)$ is not defined for $x < 0$

D. as $x \to \infty$, $f(x) \to \infty$; $f(x)$ is not defined for $x < 0$

Part B

What does the end behavior represents in the context of the situation?

Go Online to practice what you've learned in Lessons 1-1 through 1-3.

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