Linearity, Intercepts, and Symmetry

Explore Symmetry and Functions

Online Activity Use graphing technology to complete the Explore.

INQUIRY How can you tell whether the graph of a function is symmetric?

Learn Linear and Nonlinear Functions

In a **linear function**, no variable is raised to a power other than 1. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers. Linear functions can be modeled by **linear equations**, which can be written in the form $Ax + By = C$. The graph of a linear equation is a straight line.

A function that is not linear is called a **nonlinear function**. The graph of a nonlinear function includes a set of points that cannot all lie on the same line. A nonlinear function cannot be written in the form $f(x) = mx + b$. A **parabola** is a type of nonlinear function.

Example 1 Identify Linear Functions from Equations

Determine whether each function is a linear function. Justify your answer.

a. $f(x) = \frac{6x-5}{3}$

$f(x) = \frac{6x-5}{3}$	Original equation
$f(x) = \frac{6}{3}x - \frac{5}{3}$	Distribute the denominator of 3.
$f(x) = 2x - \frac{5}{3}$	Simplify.

The function ____ be written in the form $f(x) = mx + b$, so it ____ a linear function.

b. $5y = 4 + 3x^3$

$5y = 4 + 3x^3$	Original equation
$5y =$ ____ $+ 4$	Commutative Property

The function ________ be written in the form $f(x) = mx + b$ because the independent variable x is raised to a whole number power ________ 1. So, it is a ________ function.

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Today's Standards
F.IF.4; F.IF.5
MP3, MP4

Today's Vocabulary
linear function
linear equation
nonlinear function
parabola
intercept
x-intercept
y-intercept
symmetry
line symmetry
line of symmetry
point symmetry
point of symmetry
even functions
odd functions

Think About It!
Does every linear equation represent a linear function? Justify your argument.

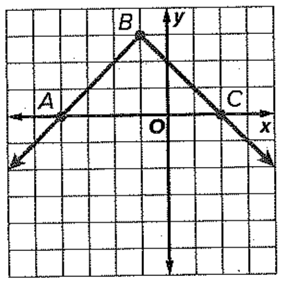
Study Tip
Linear Functions To write any linear equation in function form, solve the equation for y and replace the variable y with $f(x)$.

Your Notes

Example 2 Identify Linear Functions from Graphs

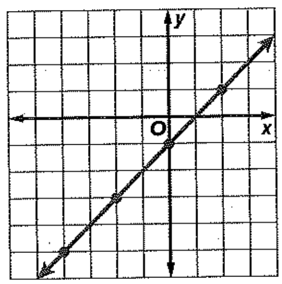
Determine whether each graph represents a *linear* or *nonlinear* function.

a.



There is no straight line that will contain the chosen points A, B, and C, so this graph represents a ________ function.

b.



The points on this graph all lie on the same line, so this graph represents a ______ function.

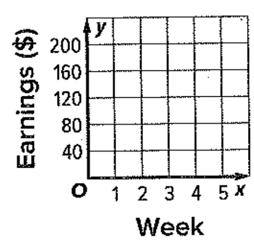
Think About It!
Are negative x- or y-values possible in the context of the situation?

Example 3 Identify Linear Functions from Tables

EARNINGS **Makayla has started working part-time at the local hardware store. Her time at work steadily increases for the first five weeks. The table shows her total earnings each of those weeks. Are her weekly earnings modeled by a *linear* or *nonlinear* function?**

Week	1	2	3	4	5
Earnings ($)	85	119	153	187	221

Graph the points that represent the week and total earnings and try to draw a line that contains all the points.



Since there is a line that contains all the points, Makaya's earning can be modeled by a ______ function.

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Learn Intercepts of Graphs of Functions

A point at which the graph of a function intersects an axis is called an **intercept**. An ***x*-intercept** is the *x*-coordinate of a point where the graph crosses the *x*-axis, and a ***y*-intercept** is the *y*-coordinate of a point where the graph crosses the *y*-axis.

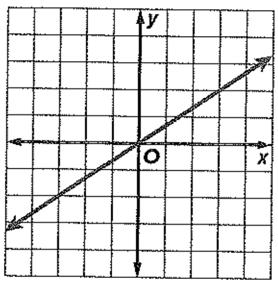
A linear function has at most one *x*-intercept while a nonlinear function may have more than one *x*-intercept.

Study Tip

Point or Coordinate *Intercept* may refer to the point or one of its coordinates. The context of the situation will often dictate which form to use.

Example 4 Find Intercepts of a Linear Function

Use the graph to estimate the *x*- and *y*-intercepts.



The graph intersects the *x*-axis at (____, ____), so the *x*-intercept is ____.

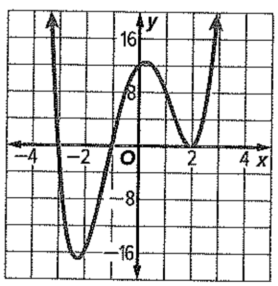
The graph intersects the *y*-axis at (____, ____), so the *y*-intercept is ____.

Think About It!

Describe a line that does not have two distinct intercepts.

Example 5 Find Intercepts of a Nonlinear Function

Use the graph to estimate the *x*- and *y*-intercepts.



The graph appears to intersect the *x*-axis at (−3, 0), (−1, 0), and (2, 0), so the functions has *x*-intercepts of ____, ____, and ____.

The graph appears to intersect the *y*-axis at (____, ____), so the function has a *y*-intercept of ____.

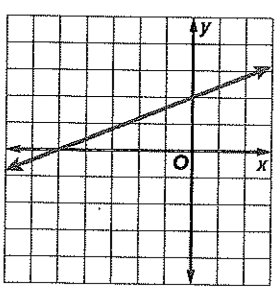
Think About It!

The graph of the nonlinear functions has three *x*-intercepts. Can the graph have more than one *y*-intercept? Explain your reasoning.

Check

Estimate the *x*- and *y*-intercepts of each graph.

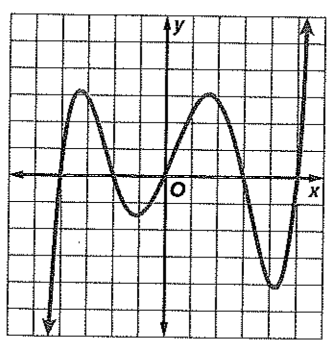
a.



x-intercept(s): ____

y-intercept(s): ____

b.



x-intercept(s): ____

y-intercept(s): ____

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Example 6 Interpret the Meaning of Intercepts

MODEL ROCKETS **Ricardo launches a rocket from a balcony. The table shows the height of the rocket after each second of its flight.**

Time (s)	Height (ft)
0	15
1	60
2	130
3	180
4	210
5	170
6	110
7	55
8	0

Part A Identify the x- and y-intercepts of the function that models the flight of the rocket.

In the table, the x-coordinate when $y = 0$ is ______. Thus, the x-intercept is ______.

In the table, the y-coordinate when $x = 0$ is ______. Thus, the y-intercept is ______.

Part B What is the meaning of the intercepts in the context of the rocket's flight?

The x-intercept is the ____________________ after the rocket is launched that it returns to the ground. The y-intercept is the ____________________ from which the rocket is launched.

Think About It!

Describe the domain of the function that models the rocket's height over time.

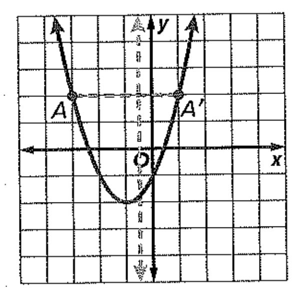
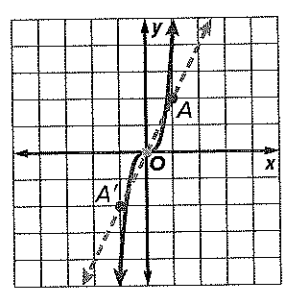
Watch Out!

Switching Coordinates A common mistake is to switch the coordinates for the intercepts. Remember that for the x-intercept, the y-coordinate is 0, and for the y-intercept, the x-coordinate is 0.

Learn Symmetry of Graphs of Functions

A figure has **symmetry** if there exists a rigid motion—reflection, translation, rotation, or glide reflection—that maps the figure onto itself.

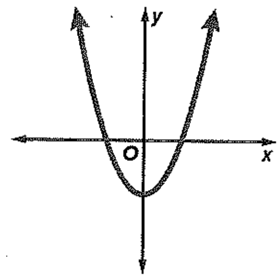
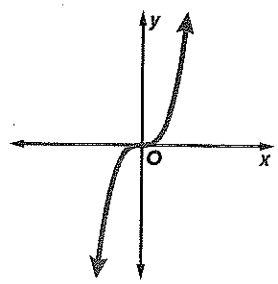
Key Concept • Symmetry

Type of Symmetry	Description	Example
A graph has **line symmetry** if each half of the graph maps exactly to the other half.	The line dividing the graph into matching halves is called the **line of symmetry**. Each point on one side is reflected in the line to a point equidistant from the line on the opposite side.	
A graph has **point symmetry** when a figure is rotated 180° about a point and maps exactly onto the other part.	The point about which the graph is rotated is called the **point of symmetry**. The image of each point on one side of the point of symmetry can be found on a line through the point of symmetry equidistant from the point of symmetry.	

Talk About It

Can the graph of a function be symmetric in a horizontal line? Justify your answer.

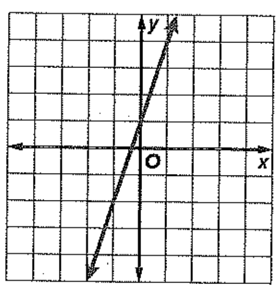
Key Concept • Even and Odd Functions

Type of Function	Algebraic Test	Example
Functions that are symmetric in the y-axis are called **even functions.**	For every x in the domain of f, $f(-x) = f(x)$.	
Functions that are symmetric about the origin are called **odd functions.**	For every x in the domain of f, $f(-x) = -f(x)$.	

Example 7 Identify Types of Symmetry

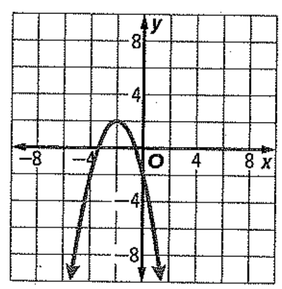
Identify the type of symmetry in the graph of each function. Explain.

a. $f(x) = 3x + 1$



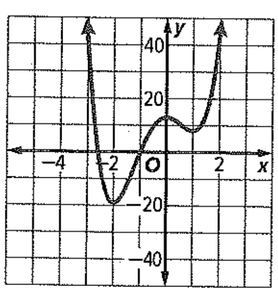
point symmetry: a 180° rotation about ________ on graph is the original graph.

b. $g(x) = -x^2 - 4x - 2$



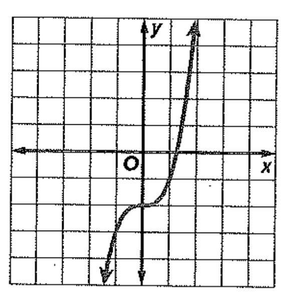
________: the reflection in the line $x = -2$ coincides with the original graph.

c. $h(x) = 3x^4 + 4x^3 - 12x^2 + 13$



________: there is no line or point of symmetry.

d. $j(x) = x^3 - 2$



point symmetry: a 180° rotation about the point ________ is the original graph.

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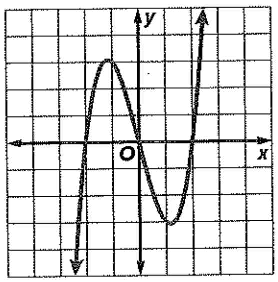
Think About It!

How would knowing the type of symmetry help you graph a function?

Example 8 Identify Even and Odd Functions

Determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If the function is odd or even, describe the symmetry.

a. $f(x) = x^3 - 4x$



It appears that the graph of $f(x)$ is symmetric about the origin. Substitute $-x$ for x to test this algebraically.

$f(-x) = (\_\_\_)^3 - 4(\_\_\_)$

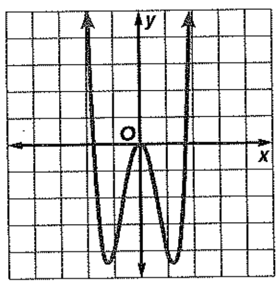
$= -x^3 + \_\_\_$ Simplify.

$= -(x^3 - 4x)$ Distribute.

$= \_\_\_$ $f(x) = x^3 - 4x$

Because $f(-x) = -f(x)$ the function is ___ and is symmetric about the ___.

b. $g(x) = 2x^4 - 6x^2$



It appears that the graph of $g(x)$ is symmetric about the y-axis. Substitute $-x$ for x to test this algebraically.

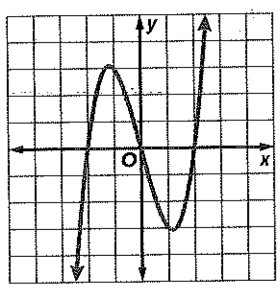
$g(-x) = 2(\_\_\_)^4 - 6(\_\_\_)^2$

$= 2x^4 - \_\_\_$ Simplify.

$= g(x)$ $g(x) = 2x^4 - 6x^2$

Because $g(-x) = g(x)$ the function is ___ and is symmetric about the ___.

c. $h(x) = x^3 + 0.25x^2 - 3x$



It appears that the graph of $h(x)$ may be symmetric about the origin. Substitute $-x$ for x to test this algebraically.

$h(-x) = (\_\_\_)^3 + 0.25(\_\_\_)^2 - 3(\_\_\_)$

$= \_\_\_ + 0.25x^2 + \_\_\_$ Simplify.

Because $-h(x) = -x^3 - 0.25x^2 + 3x$, the function is ___ because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$

Watch Out!

Even and Odd Functions Always confirm symmetry algebraically. Graphs that appear to be symmetric may not actually be.

Check

Assume that f is a function that contains the point $(2, -5)$. Which of the given points must be included in the function if f is:

even? ___ odd? ___

$(-2, -5)$ $(-2, 5)$ $(2, 5)$ $(-5, -2)$ $(-5, 2)$

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